

Magnetic forces and Particle Motion.

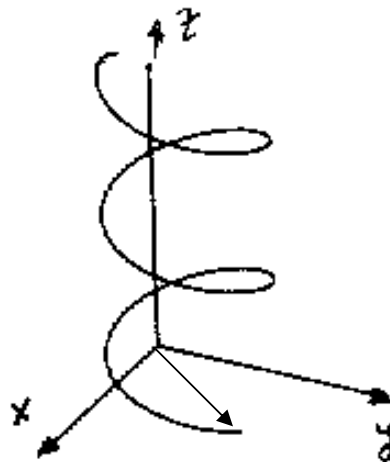
Magnetic forces are more complicated than gravity in several important ways. Unlike gravity, but similar to the ordinary Coulomb force between charged particles, magnetic forces depend on the degree to which a particle is charged. Also unlike gravity, magnetic forces possess a quality called 'polarity'. All magnets have both a north and a south 'pole'. Because of the property of polarity, the motion of charged particles in a magnetic field is more complicated than the motion under gravitational forces alone.

One common motion is for a charged particle to move in a spiral path along a line of magnetic force. As the particle moves along the field, it also executes a circular 'orbit' around the line of force, so its path resembles a helix. The spiral path can be thought of as a circular path with a radius, R , which moves at a constant speed along the line of force. Adding up the 'circular' and 'linear' motions of the particle gives you a spiral path like an unwound spring or 'Slinky' toy.

In this exercise, we will calculate the radius of such a spiral path and see how it depends on both the strength of the magnetic field, and the speed of the charged particle as it orbits the line of magnetic force. The formula for the radius of the spiral (shown by the arrow in the figure below) is given by:

$$q v B = \frac{m v^2}{R}$$

where q is the charge on the particle (in Coulombs), v is the speed of the particle as it orbits the line of force (in meters/s), B is the magnetic field strength (in Teslas), m is the mass of the particle (in kilograms), and R is the radius of the particle's orbit (in meters).



Question 1 – What is the relationship for R after solving and simplifying this equation?

Question 2 – An electron with a charge $q = 1.6 \times 10^{-19}$ Coulombs and a mass $m = 9.1 \times 10^{-31}$ kilograms is traveling at $v = 1.0 \times 10^9$ meters/sec in a magnetic field with a strength $B = 0.00005$ Teslas. What is the radius of its spiral orbit in meters?

Question 3 - If an oxygen ion has twice the electron's charge, and 29400 times an electron's mass, what will its spiral radius be for the same values of B and v in Question 1?

Question 1 - What is the relationship for R after solving and simplifying this equation?

Answer: After a little algebra:

$$q v B = \frac{m v^2}{R} \quad \text{becomes}$$

$$R = \frac{m v}{q B}$$

Question 2 – An electron with a charge $q = 1.6 \times 10^{-19}$ Coulombs and a mass $m = 9.1 \times 10^{-31}$ kilograms is traveling at $V = 1.0 \times 10^9$ meters/sec in a magnetic field with a strength $B = 0.00005$ Tesslas. What is the radius of its spiral orbit in meters?

Answer: From the equation that we just solved for R in Question 1,

$$R = (9.1 \times 10^{-31}) \times (1.0 \times 10^9) / (1.6 \times 10^{-19} \times 0.00005)$$

$$R = (9.1 \times 1.0 / (1.6 \times 5)) \times 10^{(-31 + 9 + 19 + 5)}$$

$$R = 113 \text{ meters.}$$

Question 3 - If an oxygen ion has twice the electrons charge, and 29400 times an electron's mass, what will its spiral radius be for the same values of B and V in Question 1?

Answer: The formula says that if you double the charge, the radius is decreased by $\frac{1}{2}$. If you increase the mass by 29400 times, then the radius will also increase by the same amount. So, the net change in R is $(29,400/2) = 14700$ times the electron's radius or $14700 \times 113 \text{ meters} = 1.66 \text{ million meters}$ or 1,660 kilometers. Students can solve it this way, or simply substitute into the equation for R, $B = 0.00005$, $V = 1.0 \times 10^9$, $q = 2 \times (1.6 \times 10^{-19})$, $m = 29400 \times 9.1 \times 10^{-31}$