Artist's rendition of GJ 1214b with a hypothetical moon.
(Courtesy: CfA/David Aguilar)

In December 2009, astronomers announced the discovery of the transiting super-Earth planet GJ 1214b located 42 light years from the sun, and orbits a red-dwarf star. Careful studies of this planet, which orbits a mere 2 million km from its star and takes 1.58 days to complete 'one year'. Its mass is known to be 6.5 times our Earth and a radius of about 2.7 times Earth's. Its day-side surface temperature is estimated to be 370 F, and it is locked so that only one side permanently faces its star.

When a planet passes in front of its star, light from the star passes through any atmosphere the planet might contain and travels onwards to reach Earth observers. Although the disk of the planet will temporarily decrease the brightness of its star by a few percent, the addition of an atmosphere causes an additional brightness decrease. The amount depends on the thickness of the atmosphere, the presence of dust and clouds, and the chemical composition. By studying the light dimming at many different wavelengths, astronomers can distinguish between different atmospheric constituents by using specific spectral 'fingerprints'. They can also estimate the thickness of the atmosphere in relation to the diameter of the planet.

Problem 1 – Assuming the planet is a sphere, from the available information, to two significant figures, what is the average density of the planet in kg/meter^3? (Earth mass = 6.0×10^{24} kg; Diameter = 6378 km).

Problem 2 – The average density of Earth is 5,500 kg/m^3. Suppose that GJ 1214b has a rocky core with Earth's density and a radius of R, and a thin atmosphere with a density of D. Let R = 1.0 at the surface of the planet and R=0 at the center, and assume the core is a sphere, and that the atmosphere is a spherical shell with inner radius R and outer radius R=1.0. The formula relating the atmosphere density, D, to the core radius, R, is given by:

$$1900 = (5500 - D)R^3 + D$$

A) Re-write this equation by solving for D.

B) Graph the function D(R) over the domain R:[0,1].

C) If the average density of the atmosphere is comparable to that of Venus's atmosphere for which D= 100 kg/m^3, what fraction of the radius of the planet is occupied by the Earth-like core, and what fraction is occupied by the atmosphere?

Problem 1 – Answer: Volume = 4/3 (3.141) $(2.7\text{x}6378000)^3$ = 2.1 x 10^{22} meter^3.
Density = Mass/Volume
= (6.5x6.0 x 10^{24} kg) / (2.1 x 10^{22} meter^3)
= **1,900 kg/meter^3**

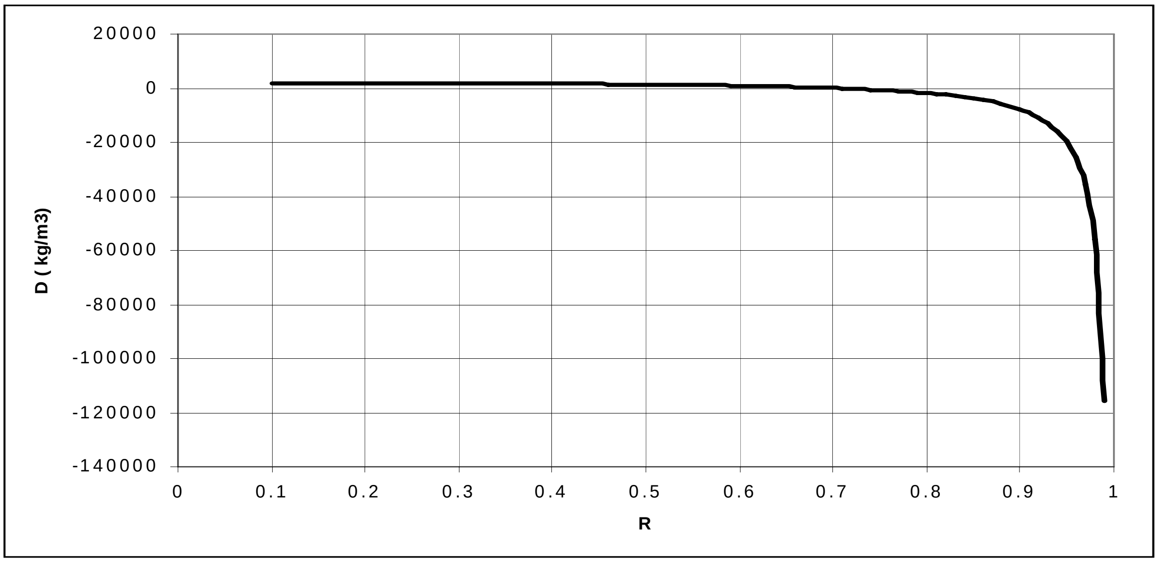
Problem 2 – A) Answer: $1900 = (5500 - D)R^3 + D$

$$1900 - D = 5500R^3 - DR^3$$

$$D\left(R^3 - 1\right) = 5500R^3 - 1900$$

$$D = \frac{5500R^3 - 1900}{R^3 - 1}$$

B) See below:



Note that for very thin atmospheres where D: [0, 100] the function predicts that the core has a radius of about R=0.7 or 70% of the radius of the planet. Since the planet's radius is 2.7 times earth's radius, the core is about 0.7x2.7 = 1.9 x earth's radius. Values for D < 0 are unphysical even though the function predicts numerical values. This is a good opportunity to discuss the limits of mathematical modeling for physical phenomena.

C) If the average density of the atmosphere is comparable to that of Venus's atmosphere for which D= 100 kg/m^3, what fraction of the radius of the planet is occupied by the Earth-like core, and what fraction is occupied by the atmosphere?

Answer: **For D=100, R = 0.71, so the core occupies the inner 71 % of the planet, and the surrounding atmospheric shell occupies the outer 29% of the planet's radius.**