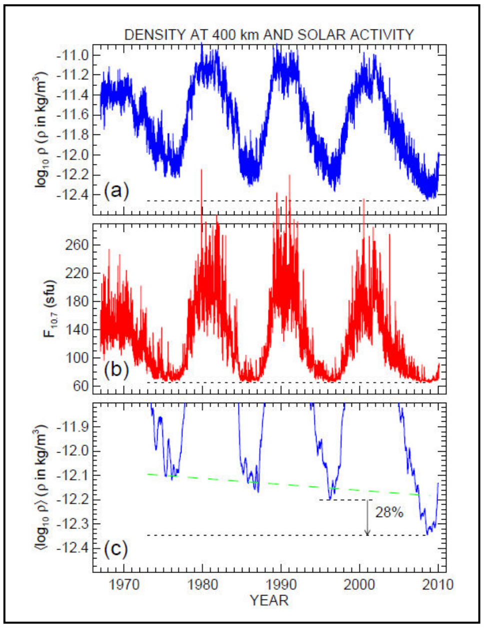

The height of Earth's thermosphere is determined, in part, by the amount of solar ultraviolet radiation reaching it, and this varies during the sunspot cycle as the graphs show. According to recent work by Dr. John Emmett at the Naval Research Laboratory, the prolonged sunspot minimum we have just passed through caused an historic shrinkage in the thermosphere density. The temperatures can range from 700K at sunspot minimum to 1,600 K at maximum. This causes the density to rise and fall as the atmosphere expands and contracts. A simple formula relates the 'scale height' of an atmosphere to its temperature:

$$H = \frac{kT}{mg}$$

In this formula, k is Boltzman's Constant, which has a value of 1.38 x10^{-23} Joules/K, g is the acceleration of gravity, which has a value of 9.8 meters/sec^2; m is the average mass of the gas particles in kilograms, and T is the temperature of the gas in Kelvins. The scale height, h, in meters represents the distance over which the gas will decrease in density by 2.7 times (a factor of e^{-1}).

Problem 1 - Suppose the gas consists of atoms of hydrogen for which m = 1.7 x 10^{-27} kg. What is the scale height, in meters for an atmosphere with a temperature of A) 700 K at sunspot minimum? B) 1,600 K at sunspot maximum?

Problem 2 - Suppose that during sunspot maximum, a satellite orbited at a constant altitude of 250 km, where the density of the gas was 1.0x10^{-12} kg/meter^3 and the temperature was 1,600 K. If the density, D, at a given altitude, z, is given by the formula

$$D(z) = D_0 e^{-\frac{Z}{H}}$$

what is the density of the atmosphere at 250 km when the temperature cools to 700 K at sunspot minimum?

Problem 1 - Suppose the gas consists of atoms of hydrogen for which m = 1.7 x 10^{-27} kg. What is the scale height, in meters for an atmosphere with a temperature of A) 700 K at sunspot minimum? B) 1,600 K at sunspot maximum?

Answer: At 700K, $H = \frac{(1.38x10^{-23})(700)}{(1.7x10^{-27})9.8}$ so **H = 580 km**

At 1,600K, H = 580 km x (1600/700) = **1,300 km**.

Problem 2 - Suppose that during sunspot maximum, a satellite orbited at a constant altitude of 250 km, where the density of the gas was 1.0x10^{-12} kg/meter^3 and the temperature was 1,600 K. If the density, D, at a given altitude, z, is given by the formula

$$D(z) = D_0 e^{-\frac{Z}{H}}$$

What is the density of the atmosphere at 250 km when the temperature cools to 700 K at sunspot minimum?

Answer: First you have to solve for D0 using H(1,600) = 1300 km and D(250) = 1.0x10^{-12} kg/meter^3 so

$$1.0x10^{-12} = D_0 e^{-\frac{250}{1300}}$$

and so D0 = 1.2x10^{-12} kg/meter^3

Now use this equation with the new H value to calculate D(250) for T = 700 K using H(700) = 580 km.

$$D(z) = 1.2x10^{-12} e^{-\frac{250}{580}}$$

so D(250) = **7.8 x 10^{-13} kg/meter^3**

So at the orbit of the satellite, during sunspot minimum, the density is lower that it was during sunspot maximum by about 100% x (0.78/1.0) = 78%.