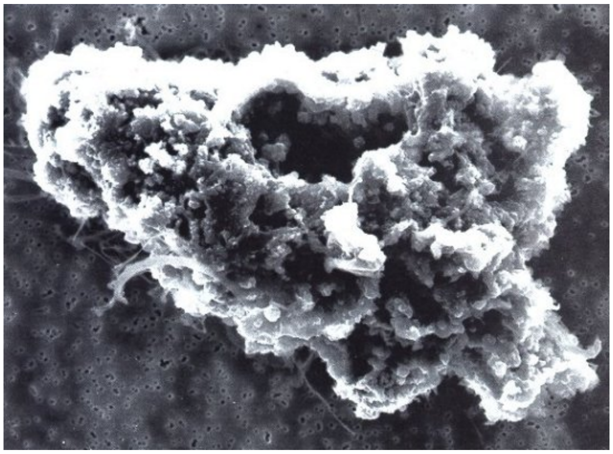
From Dust Grains to Dust Balls



A cosmic dust grain about 0.1mm across captured by a NASA high-altitude aircraft. Probably debris from a passing comet

Planets are built in several stages. The first of these involves small, micron-sized interstellar dust grains that collide and stick together to eventually form centimeter-sized bodies. A simple model of this process can tell us about how long it takes to 'grow' a rock-sized body starting from microscopic dust. This process occurs in dense interstellar clouds, which are known to be the birth places for stars and planets.

Problem 1 – Assume that the forming rock is spherical with a density of 3 gm/cc, a radius R, and a mass M. If the radius is a function of time, R(t), what is the equation for the mass of the rock as a function of time, M(t)?

Problem 2 – The rock grows by absorbing incoming dust grains that have an average mass of m grams and a density of N particles per cubic centimeter in the dust cloud. The particles collide with the surface of the rock at a speed of V cm/sec, what is the equation that gives the rate of growth of the rock's mass in time (dM/dt)?

Problem 3 – From your answer to Problem 1 and 2, re-write dM/dt in terms of M not R.

Problem 4 – Integrate your answer to problem 3 so determine M(t).

Problem 5 – What is the mass of the rock when it reaches a diameter of 1 centimeter if its density is 3 grams/cc?

Problem 6 – The rock begins at t=0 with a mass of 1 dust grain, m = 8×10^{-12} grams. The cloud density N = 3.0×10^{-5} dust grains/cc and the speed of the dust grains striking the rock, without destroying the rock, is V=10 cm/sec. How many years will the growth phase have to last for the rock to reach a diameter of 1 centimeter?

Problem 1 – Answer: Because mass = density x volume, we have
$M = 4/3\ \pi\ R^3\ \rho$ and so **$M(t) = 4/3\ \pi\ \rho\ R(t)^3$**

Problem 2 –**Answer:** The change in the mass, dM, occurs as a quantity of dust grains land on the surface area of the rock per unit time, dt. The amount is proportional to the surface area of the rock, since the more surface area the rock has, the more dust particles will be absorbed. Also, the rate at which dust grain mass is brought to the surface is proportional to the product of the dust grain density in the interstellar gas, times the speed of the grains landing on the surface. This leads to m x N x V where m is in grams per dust grain, N is in dust grains per cubic centimeter, and V is in centimeters/sec. The product of all three factors has the units of grams per square centimeter per second. The product of (m x N x V) with the surface area of the rock, will then have the units of grams/sec representing the rate at which the rock mass is growing. The full formula for the growth of the rock mass is then
$dM/dt = 4\ p\ R^2\ m\ N\ V$

Problem 3 – Answer: From Problem 1 we see that $R(t) = (3\ M(t)/ 4\ \pi\ \rho)^{1/3}$. Then substituting into dM/dt we have $dM/dt = 4\ \pi\ m\ N\ V\ (3\ M(t)/4\ \pi\ \rho)^{2/3}$ so

$$\frac{dM(t)}{dt} = 4\pi mNV\left(\frac{3}{4\pi\rho}\right)^{\frac{2}{3}} M(t)^{\frac{2}{3}}$$

Problem 4 –Answer: Re-write the differentials and move M(t) to the side with dM to get the integrand $M(t)^{-2/3}\ dM = 4\ \pi\ m\ N\ V\ (3/4\ \pi\ \rho)^{2/3}\ dt$ Then integrate both sides to get:
$3\ M(t)^{1/3} = 4\ \pi\ m\ N\ V\ (3/4\ \pi\ \rho)^{2/3}\ t + c$. Solve for M(t) to get the final equation for M(t), and remember to include the integration constant, c:

$$M(t) = \left[\frac{4}{3}\pi mNV\left(\frac{3}{4\pi\rho}\right)^{\frac{2}{3}} t + c\right]^3$$

Problem 5 –Answer: The radius will be 0.5 centimeters so, $m = 4/3\ \pi\ (0.5\ cm)^3 \times 3.0$ gm/cc = **1.6 grams.**

Problem 6 – Answer: For t = 0, M(0) = m so the constant of integration is $c = m^{1/3}$ so $c = (8\times10^{-12})^{1/3} = 2 \times 10^{-4}$.
Then $M(t) = (4/3\ (3.14)\ (8 \times 10^{-12})(3.0 \times 10^{-5})(10.0)(3/(4(3.14)\ (3.0)))^{2/3}\ t + 2 \times 10^{-4})^3$
$M(t) = (1.9 \times 10^{-15}\ t + 2 \times 10^{-4})^3$

To reach M(t) = 1.6 grams, $t = 6.1 \times 10^{14}$ seconds or about **19 million years!**