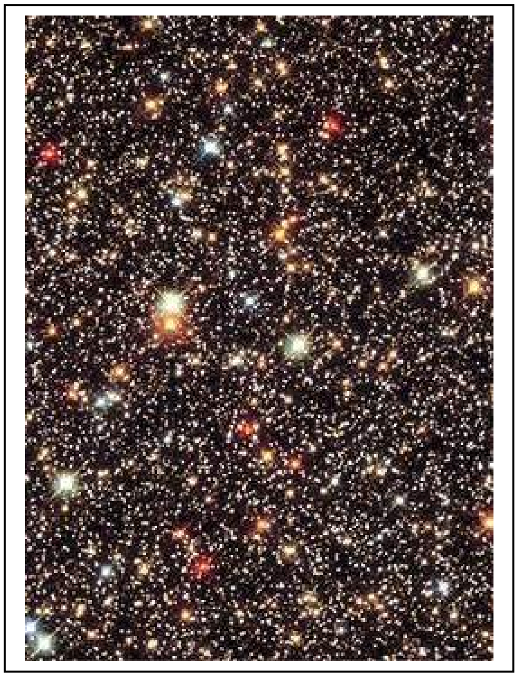
How many stars are in the sky?



One of the very first things that astronomers studied was the number of stars in the sky. From this, they hoped to get a mathematical picture of the shape and extent of the entire Milky Way galaxy. This is perhaps why some cartoons of 'astronomers' often have them sitting at a telescope and tallying stars on a sheet of paper! Naked-eye counts usually number a few thousand, but with increasingly powerful telescopes, fainter stars can be seen and counted, too.

Over the decades, 'star count' sophisticated models have been created, and rendered into approximate mathematical functions that let us explore what we see in the sky. One such approximation, which gives the average number of stars in the sky, is shown below:

$$\mathrm{Log}_{10}N(m) = -0.0003\ m^3 + 0.0019\ m^2 + 0.484\ m - 3.82$$

This polynomial is valid over the range [+4.0, +25.0] and gives the Log_{10} of the total number of stars per square degree fainter than an apparent magnitude of m. For example, at an apparent magnitude of +6.0, which is the limit of vision for most people, the function predicts that $\mathrm{Log}_{10}N(6) = -0.912$ so that there are $10^{-0.912} = 0.12$ stars per square degree of the sky. Because the full sky area is 41,253 square degrees, there are about 5,077 stars brighter than, or equal to, this magnitude.

Problem 1 - A small telescope can detect stars as faint as magnitude +10. If the human eye-limit is +6 magnitudes, how many more stars can the telescope see than the human eye?

Problem 2 - The Hubble Space Telescope can see stars as faint as magnitude +25. About how many stars can the telescope see in an area of the sky the size of the full moon (1/4 square degree)?

Problem 3 - A photograph is taken of a faint star cluster that has an area of 1 square degree. If the astronomer counts 5,237 stars in this area of the sky with magnitudes in the range from +11 to +15, how many of these stars are actually related to the star cluster?

Problem 1 - Answer: From the example, there are 0.12 stars per square degree brighter than +6.0

$$\begin{aligned} Log_{10}N(+10) &= -0.0003\,(10)^3 + 0.0019\,(10)^2 + 0.484\,(10) - 3.82 \\ &= -0.3 + 0.19 + 4.84 - 3.82 \\ &= +0.55 \end{aligned}$$

So there are $10^{.55}$ = 3.55 stars per square degree brighter than +10. Converting this to total stars across the sky (area = 41,253 square degrees) we get 5,077 stars brighter than +6 and 146,448 stars brighter than +10. The number of additional stars that the small telescope will see is then 146,448 - 5,077 = **141,371 stars.**

Problem 2 - Answer: $Log_{10}N(25) = -0.0003\,(25)^3 + 0.0019\,(25)^2 + 0.484\,(25) - 3.82$
$= +4.78$

So the number of stars per square degree is $10^{+4.78}$ = 60,256. For an area of the sky equal to 1/4 square degree we get (60,256) x (0.25) = **15,064 stars**.

Problem 3 - Answer: $Log_{10}N(15)$ counts up all of the stars in an area of 1 square degree with magnitudes of 0, +1, +2,…+15. $Log_{10}N(10)$ counts up all of the stars in an area of 1 square degree with magnitudes of 0, +1, +2,…+10. The difference between these two will be the number of stars with magnitudes of +11, +12, +13, +15, which is the number of stars in the sky per square degree, in the magnitude range of the star cluster. We then subtract this number from 5,237 to get the number of stars actually in the cluster.

$Log_{10}N(15) = +2.86$ corresponds to $10^{+2.86}$ = 716.1 stars/square degree

$Log_{10}N(11) = +1.33$ corresponds to $10^{+1.33}$ = 21.6 stars/square degree

So the difference between these is 716.1-21.6 = 694.5 stars/deg^2. The star cluster area is 5 square degrees, so we have 694.5 stars/deg^2 x (5 square degrees) = 3,473 stars that we expect to find in the sky in the magnitude range of the star cluster. Since the astronomer counted 5,237 stars in the star cluster field, that means that 3,473 of these stars are probably just part of the general sky population of stars, while only 5,237 - 3,473 = **1,764 stars are actually members of the cluster** itself.

Note to Teacher: Show that the students have to evaluate N first before taking the difference because $Log_{10}N(15) - Log_{10}N(11)$ is not the same as N(15) - N(10).