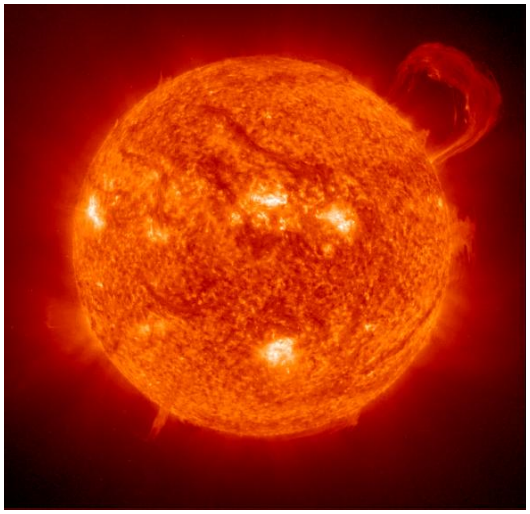

Detailed mathematical models of the interior of the sun are based on astronomical observations and our knowledge of the physics of stars. These models allow us to explore many aspects of how the sun 'works' that are permanently hidden from view.

The Standard Model of the sun, created by astrophysicists during the last 50 years, allows us to investigate many separate properties. One of these is the density of the heated gas throughout the interior. The function below gives a best-fit formula, D(x) for the density (in grams/cm^3) from the core (x=0) to the surface (x=1) and points in-between.

$$\boxed{D(x) = 519x^4 - 1630x^3 + 1844x^2 - 889x + 155}$$

For example, at a radius 30% of the way to the surface, x = 0.3 and so D(x=0.3) = 14.5 grams/cm^3.

Problem 1 - What is the estimated core density of the sun?

Problem 2 - To the nearest 1% of the radius of the sun, at what radius does the density of the sun fall to 50% of its core density at x=0? (Hint: Use a graphing calculator and estimate x to 0.01)

Problem 3 - What is the estimated density of the sun near its surface at x=0.9 using this polynomial approximation?

Problem 4 - Integrate D(x) throughout the volume of the solar interior to estimate the total mass of the sun in grams. (Use the volume element dV = $4\pi x^2$ dx, and use the fact that for x=1, the physical radius of the sun is 6.9×10^{10} centimeters.)

Problem 1 - Answer; At the core, x=0, do D(0) = 155 grams/cm^3.

Problem 2 - Answer: We want D(x) = 155/2 = 77.5 gm/cm3. Use a graphing calculator, or an Excell spreadsheet, to plot D(x) and slide the cursor along the curve until D(x) = 77.5 .Then read out the value of x. The relevant portion of D(x) is shown in the table below:

X	D(x)
0.08	94.87
0.09	88.77
0.1	82.96
0.11	**77.43**
0.12	72.16
0.13	67.16
0.14	62.41

Problem 3 - Answer: At x=0.9 (i.e. a distance of 90% of the radius of the sun from the center).
D(0.9) = $519(0.9)^4 - 1630(0.9)^3 + 1844(0.9)^2 - 889(0.9) + 155$
D(0.9) = 340.516 - 1188.27 + 1493.64 - 800.10 + 155.00
D(0.9) = 0.786 gm/cm^3.

Problem 4 - Integrate D(x) throughout the volume of the solar interior to estimate the total mass of the sun in grams. (Use the volume element dV = $4\pi x^2$ dx and use the fact that for x=1, the physical radius of the sun is 6.9×10^{10} centimeters.). Answer:

$$M = \int_0^1 [519x^4 - 1630x^3 + 1844x^2 - 889x + 155] 4\pi x^2 dx$$

$$M(x) = 4\pi[\frac{519}{7}x^7 - \frac{1630}{6}x^6 + \frac{1844}{5}x^5 - \frac{889}{4}x^4 + \frac{155}{3}x^3]$$

$$M(1) = 4(3.14)[74.14 - 271.67 + 368.80 - 222.25 + 51.67]$$

$$M(1) = 4(3.14)[0.69]$$

$$M(1) = 8.71$$

Since x=1 is a physical distance of R = 6.9×10^{10} centimeters, we have to multiply M(1) by $(6.9 \times 10^{10})^3$ to get M = 8.29 x (3.28×10^{32} cm^3) or M = 2.7×10^{33} grams. The actual astronomical value is 1.98×10^{33} grams. The polynomial approximation for the internal density is a close, but not exact, estimate.