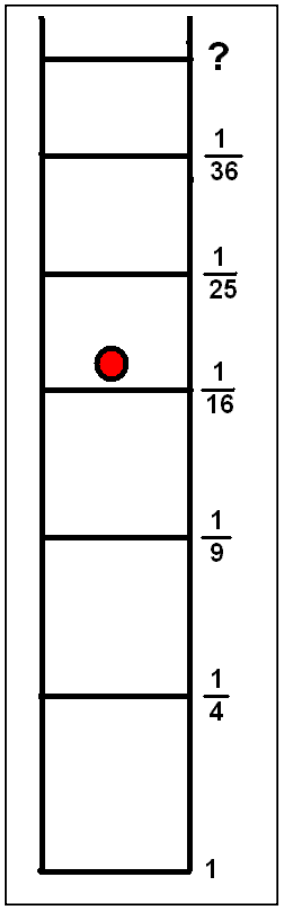
Atomic Fractions



The single electron inside a hydrogen atom can exist in many different energy states. The lowest energy an electron can have is called the Ground State: this is the bottom rung on the ladder marked with an energy of '1'.

The electron must obey the Ladder Rule. This rule says that the electron can gain or lose only the exact amount of energy defined by the various ladder intervals.

For example, if it is located on the third rung of the ladder marked with an energy of '1/9', and it loses enough energy to reach the Ground State, it has to lose exactly 1 - 1/9 = 8/9 units of energy.

The energy that the electron loses is exactly equal to the energy of the light that it emits. This causes the spectrum of the atom to have a very interesting 'bar code' pattern when it is sorted by wavelength like a rainbow. The 'red line' is at a wavelength of 656 nanometers and is caused by an electron jumping from Energy Level 3 to Energy Level 2 on the ladder.

To answer these questions, use the Energy Fractions in the above ladder, and write your answer as the simplest fraction. Do not use a calculator or work with decimals because these answers will be less-exact than leaving them in fraction form!

Problem 1 - To make the red line in the spectrum, how much energy did the electron have to lose on the energy ladder?

Problem 2 - How much energy will the electron have to gain (+) or lose (-) in making the jumps between the indicated rungs:
A) Level-2 to Level-5
B) Level-3 to Level-1
C) Level-6 to Level-4
D) Level-4 to Level-6
E) Level-2 to Level-4
F) Level-5 to Level-1
G) Level-6 to Level-5

Problem 3 - From the energy of the rungs in the hydrogen ladder, use the pattern of the energy levels (1, 1/4, 1/9, 1/16, 1/25, …) to predict the energy of the electron jumping from A) the 10th rung to the 7th rung; B) the rung M to the lower rung N.

Problem 4 - If an energy difference of '1' on the ladder equals an energy of 14 electron-Volts, in simplest fractional form, how many electron-Volts does the electron lose in jumping from Level-6 to Level-4?

Answer Key

Problem 1 - Answer: The information in the figure says that the electron jumped from Level-3 to Level-2. From the energy ladder, this equals a difference of 1/9 - 1/4. The common denominator is '36' so the fractions become 4/36 - 9/36 and the difference is -5/36 .because the answer is negative, the electron has to **lose 5/36** of a unit of energy to make the jump.

Problem 2 - How much energy will the electron have to gain (+) or lose (-) in making the jumps between the indicated rungs:
A) Level-2 to Level-5 = 1/4 - 1/25 = (25 - 4)/100 = **+21/100** so it has to GAIN energy.
B) Level-3 to Level-1 = 1/9 - 1 = 1/9 - 9/9 = **-8/9** so it has to LOSE energy.

C) Level-6 to Level-4 = 1/36 - 1/16 = **-5/144** so it has to LOSE energy
Two ways to solve:
First: (16 - 36) / (16x36) = -20 / 576 then simplify to get - 5 / 144
Second: Find Least Common Multiple
36: 36, 72, 108, **144**, 180, …
16: 16, 32, 48, 64, 80, 96, 112, 128, **144**, 160, …
LCM = 144, then
1/36 - 1/16 = 4/144 - 9/144 = **-5/144**

D) Level-4 to Level-6 = 1/16 - 1/36 = **+5/144** so it has to Gain energy.
E) Level-2 to Level-4 = 1/4 - 1/16 = 4/16 - 1/16 = **+3/16** so it has to GAIN energy
F) Level-5 to Level-1 = 1/25 - 1 = 1/25 - 25/25 = **-24/25** so it has to LOSE energy
G) Level-6 to Level-5 = 1/36 - 1/25 = (25 - 36)/ 900 = **- 11/900** so it has to LOSE energy

Problem 3 - Answer: Students should be able to see the pattern from the series progression such that the energy is the reciprocal of the square of the ladder rung number.

Level 2 Energy = $1/(2)^2 = 1/4$
Level 5 Energy = $1/(5)^2 = 1/25$

A) the 10th rung to the 7th rung: Energy = 1/100 - 1/49 = (49 - 100)/4900 = **-51/4900**.

B) the rung M to the lower rung N. Energy = $1/M^2 - 1/N^2$

Problem 4 - If an energy difference of '1' on the ladder equals an energy of 13.6 electron-Volts, in simplest fractional form how many electron-Volts does the electron lose in jumping from Level-6 to Level-4?

Answer; The energy difference would be 1/36 - 1/16 = **-5/144** energy units.

Since an energy difference of 1.0 equals 14 electron-Volts, by setting up a ratio we have:

$$\frac{5/144 \text{ Units}}{1 \text{ Unit}} = \frac{X}{14 \text{ eV}} \quad \text{so} \quad X = 14 \times (5/144) = \frac{5 \times 2 \times 7}{2 \times 72} = \mathbf{\frac{35}{72}\ eV}$$