Finding Mass in the Cosmos

$$Fg = \frac{G\,M\,m}{R^2}$$

$$Fc = \frac{m\,V^2}{R}$$

$$V = \frac{2\pi R}{T}$$

One of the neatest things in astronomy is being able to figure out the mass of a distant object, without having to 'go there'. Astronomers do this by employing a very simple technique. It depends only on measuring the separation and period of a pair of bodies orbiting each other. In fact, Sir Issac Newton showed us how to do this over 300 years ago!

Imagine a massive body such as a star, and around it there is a small planet in orbit. We know that the force of gravity, **Fg**, of the star will be pulling the planet inwards, but there will also be a centrifugal force, **Fc**, pushing the planet outwards.

This is because the planet is traveling at a particular speed, **V**, in its orbit. When the force of gravity and the centrifugal force on the planet are exactly equal so that **Fg = Fc**, the planet will travel in a circular path around the star with the star exactly at the center of the orbit.

Problem 1) Use the three equations above to derive the mass of the primary body, M, given the period, T, and radius, R, of the companion's circular orbit.

Problem 2) Use the formula $\mathbf{M = 4\pi^2 R^3 / (G\,T^2)}$ where $G = 6.6726 \times 10^{-11}$ N-m^2/kg^2 and M is the mass of the primary in kilograms, R is the orbit radius in meters and T is the orbit period in seconds, to find the masses of the primary bodies in the table below. (Note: Make sure all units are in meters and seconds first! 1 light years = 9.5 trillion kilometers)

Primary	Companion	Period	Orbit Radius	Mass of Primary
Earth	Communications satellite	24 hrs	42,300 km	
Earth	Moon	27.3 days	385,000 km	
Jupiter	Callisto	16.7 days	1.9 million km	
Pluto	Charon	6.38 days	17,530 km	
Mars	Phobos	7.6 hrs	9,400 km	
Sun	Earth	365 days	149 million km	
Sun	Neptune	163.7 yrs	4.5 million km	
Sirius A	Sirius B	50.1 yrs	20 AU	
Polaris A	Polaris B	30.5 yrs	290 million miles	
Milky Way	Sun	225 million yrs	26,000 light years	

Answer Key

Problem 1: Answer

$$\frac{G\,M\,m}{R^2} = \frac{m\,V^2}{R}$$

Cancil the companion mass, m, on both sides, and isolate the primary mass, M, on the left side:

$$M = \frac{R\,V^2}{G}$$

Now use the definition of V to eliminate it from the equation,

$$M = \frac{R}{G}\left(\frac{2\pi R}{T}\right)^2$$

and simplify

$$M = \frac{4\pi^2 R^3}{G\,T^2}$$

Problem 2:

Primary	Companion	Period	Orbit Radius	Mass of Primary
Earth	Communications satellite	24 hrs	42,300 km	**6.1×10^{24} kg**
Earth	Moon	27.3 days	385,000 km	**6.1×10^{24} kg**
Jupiter	Callisto	16.7 days	1.9 million km	**1.9×10^{27} kg**
Pluto	Charon	6.38 days	17,530 km	**1.3×10^{22} kg**
Mars	Phobos	7.6 hrs	9,400 km	**6.4×10^{23} kg**
Sun	Earth	365 days	149 million km	**1.9×10^{30} kg**
Sun	Neptune	163.7 yrs	4.5 million km	**2.1×10^{30} kg**
Sirius A	Sirius B	50.1 yrs	298 million km	**6.6×10^{30} kg**
Polaris A	Polaris B	30.5 yrs	453 million km	**6.2×10^{28} kg**
Milky Way	Sun	225 million yrs	26,000 light years	**1.7×10^{41} kg**

Note: The masses for Sirius A and Polaris A are estimates because the companion star has a mass nearly equal to the primary so that our mass formula becomes less reliable.