The Limiting Behavior of Selected Functions

In astrophysics, many different kinds of formulae are derived for the interaction of radiation with matter. Often, astrophysicists want to know the 'limiting behavior' of the equations for extreme conditions. Here are a few examples.

$$\sigma = \frac{3}{4} S \left(\frac{1+x}{x^2} \left[\frac{2(1+x)}{1+2x} - \frac{1}{x} \ln(1+2x) \right] + \frac{1}{2x} \ln(1+2x) - \frac{1+3x}{(1+2x)^2} \right)$$

Problem 1 : The equation above is the Klien and Nishina formula for the interaction of a high-energy photon with an electron. X is the ratio of the energy carried by the photon ($E = hv$) compared to the rest mass energy of the electron ($E = mc^2$). What is the form of this equation in the limit for large X?

Problem 1 – The formula can be simplified by noticing that as X becomes very large compared to 1, (1 + x) becomes x, and (1 + 2x) becomes 2x, (1 + 3x) becomes 3x, and the formula simplifies to

$$\sigma = 3/4\ s\ (1/x)[\ 2x/2x - (1/x)\ln(2x)] + (1/2x)\ln(2x) - 3x/(2x)^2).$$

This becomes $\sigma = 3/4\ s\ (1/x - (1/x^2)\ln(2x) + (1/2x)\ln(2x) - 3/4x)$

Because terms involving $1/x^2$ diminish faster than terms with 1/x, this leaves us with

$$\sigma = 3/4\ s\ [(1/2x)\ln(2x) + 1/4x]$$

which further simplifies to

$$\mathbf{\sigma = (3s/8x)[\ ln(2x) + 1/2\ ]}$$