Collapsing Gas Clouds - Stability

Gas clouds in interstellar space are acted upon by external pressure and their own gravity, and would otherwise collapse, but if they are hot enough, they can remain stable for a long time. That seems to be the case for objects called Bok Globules.

This photo of Thackeray's Globule (IC-2944) taken by the Hubble Space Telescope may be a stable dark cloud containing 10 times the mass of our sun at a temperature of less than 100 K.

A gas sphere with a radius, R, a mass, M, and a temperature, T, is subject to an external pressure, P so that

$$P = \frac{3MkT}{4\pi R^3 \mu m} - \frac{3GM^2}{20\pi R^4}$$

where k, G and μ are constants.

Problem 1 - At a given external temperature, P, the cloud will be in equilibrium and not collapse if the sum of the two internal pressure terms on the right-side of the equation add up to the same value as the external pressure.

At what minimum radius will the cloud start to collapse for a given mass and temperature because the internal pressure no longer equals the external pressure? (Hint: determine dP/dR and find minimum.

Problem 1 - The problem states that the mass and temperature are held constant, so the only free variable is R. For complicated equations, it is always a good idea to group all constants together and define new constants. You can later replace the new constants by the old ones. Let's define $A = (3MkT/4\pi\mu m)$ and $B=(3GM^2/20\pi)$, then the equation becomes $P = AR^{-3} - BR^{-4}$. To find the extremum, we calculate dP/dR and set this equal to zero. This gives us $dP/dR = A(-3)R^{-4} - B(-4)R^{-5} = 0$. This leads to R = 4 B/3 A which upon substituting back for the definitions of A and B gives us

$$R_c = \frac{12G\mu m}{45kT}$$