

The Oscillation Period of Gaseous Spheres

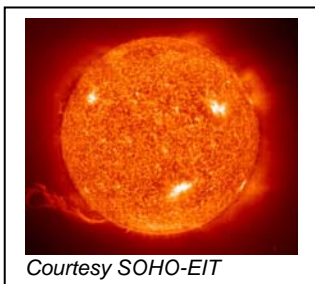
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Any collection of matter that is governed by the force of gravity has a natural period of oscillation. For example, a simple pendulum such as a playground swing, will move back and forth with a time period in seconds, T , given by the formula to the right, where L is the length of the swing in centimeters, and g is the acceleration of gravity at Earth's surface given by 980 cm/sec^2 . The period of a swing that is 3 meters long is then $T = 3.5$ seconds.

$$T = 2\pi \sqrt{\frac{L}{g}}$$

This behavior also applies to any body held together by gravity whether it is a star or a planet. The natural oscillation period of such bodies is given by the formula to the right, where D is the density of the body in grams/cm^3 and G is the Newtonian constant of Gravity $G = 6.6 \times 10^{-8} \text{ dynes cm}^2 \text{ gm}^{-2}$ and T is in seconds. For example, the planet Jupiter has a density of about $D = 1.3 \text{ gm/cm}^3$ so its period, T , will be about 10,500 seconds or $T = 3$ hours. From the information below, calculate the natural periods for the various astronomical bodies.

$$T^2 = \frac{3\pi}{GD}$$



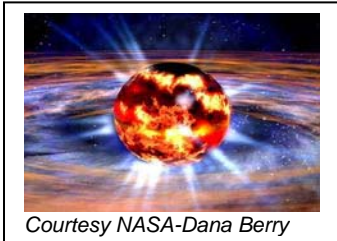
The sun is about 1.5 million kilometers across, and has an average density of about 1.5 grams/cm^3

$T =$ _____ hours.



The Earth has a diameter of about 12,500 kilometers, and has an average density of about 5.5 grams/cm^3

$T =$ _____ minutes.



A neutron star is about 50 kilometers in diameter, and has an average density of about $2 \times 10^{14} \text{ grams/cm}^3$

$T =$ _____ seconds.

Answer Key:

The sun is about 1.5 million kilometers across, and has an average density near its surface of about 1.5 grams/cm^3

$$\begin{aligned} T^2 &= 3 \times (3.141) / (1.5 \times 6.6 \times 10^{-8}) \\ &= 9.5 \times 10^7 \end{aligned}$$

$$\begin{aligned} \text{So } T &= 9,800 \text{ seconds} \\ &= 2.7 \text{ hours} \end{aligned}$$

The Earth has a diameter of about 12,500 kilometers, and has an average density of about 5.5 grams/cm^3

$$\begin{aligned} T^2 &= 3 \times (3.141) / (5.5 \times 6.6 \times 10^{-8}) \\ &= 2.5 \times 10^7 \end{aligned}$$

$$\begin{aligned} \text{So } T &= 5,100 \text{ seconds} \\ &= 85 \text{ minutes} \end{aligned}$$

A neutron star is about 50 kilometers in diameter, and has an average density of about $2 \times 10^{14} \text{ grams/cm}^3$

$$\begin{aligned} T^2 &= 3 \times (3.141) / (2.0 \times 10^{14} \times 6.6 \times 10^{-8}) \\ &= 7.1 \times 10^{-7} \end{aligned}$$

$$\text{So } T = 0.00085 \text{ seconds}$$