



Outside a black hole, we have the normal universe of space, time, matter and energy we have all come to know. But inside, things are very different. We know this because the same mathematics that predicts black holes should exist, also predicts what to find inside them. One of the biggest surprises is the way that time and space, themselves, behave.

Inside a black hole, the roles played by time and space reverse themselves. Before we are crushed to death by the Singularity, we have limited freedom to intentionally move through space, but some freedom in how we travel through time...in our last moments of life!

Problem 1 - In relativity, space and time are part of a single 4-dimensional thing called spacetime. There are 3 dimensions to space and 1-dimension to time. Every point, called an Event, has three coordinates to describe its location in space, and one extra coordinate to describe its location in time. We write these as an ordered set like $A(x,y,z,t)$ or $B(x,y,z,t)$. Write the ordered set for the following event called A: I travel north 5 miles, east 3 miles, and up 1 miles, at 9:00AM on February 16, 2008.

Problem 2 - Suppose I travel from one event with coordinates $A(3 \text{ km}, 6 \text{ km}, 2 \text{ km}, 5:00\text{PM})$ to another event $B(5 \text{ km}, 7 \text{ km}, 3 \text{ km}, 8:00\text{PM})$. How far did I travel in space during the time interval from 5:00 PM to 8:00 PM?

Problem 3 - Use the Pythagorean Theorem to calculate the actual distance in space in Problem 2.

Problem 4 - The 4-dimensional, 'hyper' distance between the events is found by using the 'hyperbolic' Pythagorean Theorem formula $D^2 = -c^2T^2 + X^2 + y^2 + z^2$ where c is the speed of light ($c = 300,000 \text{ km/sec}$). Calculate the hyper-distance, D^2 , between Events A and B in Problem 2.

Problem 5 - Based on your answer, which part of D^2 makes the largest contribution to the hyper-distance, the time-like part, T , or the space-like part (x,y,z) ?

Problem 6 - Inside a black hole, the formula for D^2 changes to $D^2 = c^2T^2 - x^2 - y^2 - z^2$. Suppose Events A and B are now happening inside the black hole. What is the hyper-distance between them?

Extra for Experts:

Problem 7 - If an observer defines 'time' as the part of D^2 that has a negative sign, and 'space' as the part that has the positive sign, can you explain what happens as the traveler passes inside the black hole?

Answer Key:

Problem 1 - In relativity, space and time are part of a single 4-dimensional thing called spacetime. There are 3 dimensions to space and 1-dimension to time. Every point, called an Event, has three coordinates to describe its location in space, and one extra coordinate to describe its location in time. We write these as an ordered set like A(x,y,z,t) or B(x,y,z,t). Write the ordered set for the following event called A: I travel north 5 miles, east 3 miles, and up 1 miles, at 9:00AM on February 16, 2008.

Answer - **A(5,3,1, 9:00AM 2/16/2008)**

Problem 2 - Suppose I travel from one event with coordinates A(3 km, 6 km, 2 km, 5:00PM) to another event B(5 km, 7 km, 3 km, 8:00PM). How far did I travel in space during the time interval from 5:00 PM to 8:00 PM?

Answer: Take the difference in the x, y and z coordinates to get $x = 5-3 = 2$ km; $y=6-3 = 3$ km and $z=3-2 = 1$ km.

Problem 3 - Use the Pythagorean Theorem to calculate the actual distance in space in Problem 2.

Answer: This only involves the x, y and z coordinate differences we found in Problem 2: Distance = $(2^2 + 3^2 + 1^2)^{1/2} = (14)^{1/2}$ or **3.7 kilometers**.

Problem 4 - The 4-dimensional, 'hyper' distance between the events is found by using the 'hyperbolic' Pythagorean Theorem formula $D^2 = -c^2T^2 + X^2 + y^2 + z^2$ where c is the speed of light ($c = 300,000$ km/sec). Calculate the hyper-distance, D^2 , between Events A and B in Problem 2.

Answer: First, the time difference is 8:00PM - 5:00 PM = 3 hours. This equals 10,800 seconds. Then from the formula, where all units are in kilometers, we get $D^2 = -(300,000 \text{ km/sec})^2 (10,800 \text{ sec})^2 + 2^2 + 3^2 + 1^2 = -1.05 \times 10^{19}$ kilometers.

Problem 5 - Based on your answer, which part of D^2 makes the largest contribution to the hyper-distance, the time-like part, T, or the space-like part (x,y,z)?

Answer - **D^2 is negative, so it is the time-like part that makes the biggest difference.**

Problem 6 - Inside a black hole, the formula for D^2 changes to $D^2 = c^2T^2 - X^2 - y^2 - z^2$. Suppose Events A and B are now happening inside the black hole. What is the hyper-distance between them?

Answer - $D^2 = (300,000 \text{ km/sec})^2 (10,800 \text{ sec})^2 - 2^2 - 3^2 - 1^2 = +1.05 \times 10^{19}$ kilometers.

Problem 7 - If an observer defines 'time' as the part of D^2 that has a negative sign, and 'space' as the part that has the positive sign, can you explain what happens as the traveler passes inside the black hole?

Answer - **Outside the black hole, the T variable we defined to be time is part of the negative-signed component to D^2 , and x,y,z are the space variables and are the positive part of D^2 . When we enter the black hole, the T variable becomes part of the positive space-like part of D^2 , and x, y and z are part of the negative part of D^2 . This means that the roles of space and time have been reversed inside a black hole!**