



Black holes are objects that have such intense gravitational fields, they do not allow light to escape from them. They also make it impossible for anything that falls into them to escape, because to do so, they would have to travel at speeds faster than light. No forms of matter or energy can travel faster than the speed of light, so that is why black holes are so unusual!

### There are three parts to a simple black hole:

**Event Horizon** - Also called the Schwarzschild radius, that's the part that we see from the outside. It looks like a black, spherical surface with a very sharp edge in space.

**Interior Space** - This is a complicated region where space and time can get horribly mangled, compressed, stretched, and otherwise a very bad place to travel through.

**Singularity** - That's the place that matter goes when it falls through the event horizon. It's located at the center of the black hole, and it has an enormous density. You will be crushed into quarks long before you get there!

Black holes can, in theory, come in any imaginable size. The size of a black hole depends on the amount of mass it contains. It's a very simple formula, especially if the black hole is not rotating. These 'non-rotating' black holes are called Schwarzschild Black Holes.

Equation 1) 
$$R = \frac{2GM}{c^2}$$

Equation 2) 
$$R = 1.48 \times 10^{-27} M$$

**Problem 1** - The two formulas above give the Schwarzschild radius, R, of a black hole in terms of its mass, M. From Equation 1, verify Equation 2, which gives R in meters and M in kilograms, using  $c = 3 \times 10^8$  m/sec for the speed of light, and  $G = 6.67 \times 10^{-11}$  Newtons  $m^2/kg^2$  for the gravitational constant.

**Problem 2** - Calculate the Schwarzschild radius, in meters, for Earth where  $M = 5.9 \times 10^{24}$  kilograms.

**Problem 3** - Calculate the Schwarzschild radius, in kilometers, for the sun, where  $M = 1.9 \times 10^{30}$  kilograms.

**Problem 4** - Calculate the Schwarzschild radius, in kilometers, for the entire Milky Way, with a mass of 250 billion suns.

**Problem 5** - Calculate the Schwarzschild radius, in meters, for a black hole with the mass of an average human being with  $M = 60$  kilograms.

## Answer Key

**Problem 1** - The two formulas above give the Schwarzschild radius,  $R$ , of a black hole in terms of its mass,  $M$ . From Equation 1, verify Equation 2, which gives  $R$  in meters and  $M$  in kilograms, using  $c = 3 \times 10^8$  m/sec for the speed of light, and  $G = 6.67 \times 10^{-11}$  Newtons  $m^2/kg^2$  for the gravitational constant.

$$\begin{aligned} \text{Answer: Radius} &= 2 \times (6.67 \times 10^{-11}) / (3 \times 10^8)^2 M \text{ meters} \\ &= \mathbf{1.48 \times 10^{-27} M \text{ meters}} \end{aligned}$$

where  $M$  is the mass of the black hole in kilograms.

**Problem 2** - Calculate the Schwarzschild radius, in meters, for Earth where  $M = 5.9 \times 10^{24}$  kilograms.

$$\begin{aligned} \text{Answer: } R &= 1.48 \times 10^{-27} (5.9 \times 10^{24}) \text{ meters} \\ \mathbf{R} &= \mathbf{0.0088 \text{ meters or } 8.8 \text{ millimeters}} \end{aligned}$$

**Problem 3** - Calculate the Schwarzschild radius, in kilometers, for the sun, where  $M = 1.9 \times 10^{30}$  kilograms.

$$\begin{aligned} \text{Answer: } R &= 1.48 \times 10^{-27} (1.9 \times 10^{30}) \text{ meters} \\ \mathbf{R} &= \mathbf{2.8 \text{ kilometers}} \end{aligned}$$

**Problem 4** - Calculate the Schwarzschild radius, in kilometers, for the entire Milky Way, with a mass of 250 billion suns.

Answer: If a black hole with the mass of the sun has a radius of 2.8 kilometers, a black hole with 250 billion times the sun's mass will be 250 billion times larger, or

$$R = (2.8 \text{ km} / \text{sun}) \times 250 \text{ billion suns} = \mathbf{700 \text{ billion kilometers.}}$$

Note: The entire solar system has a radius of about 4.5 billion kilometers!

**Problem 5** - Calculate the Schwarzschild radius, in meters, for a black hole with a mass of an average human being with  $M = 60$  kilograms.

$$\begin{aligned} \text{Answer: } R &= 1.48 \times 10^{-27} (60) \text{ meters} \\ \mathbf{R} &= \mathbf{8.9 \times 10^{-26} \text{ meters.}} \end{aligned}$$

Note: A proton is only about  $10^{-16}$  meters in diameter.