The Volume of a Hypersphere

This spherical propellant tank is an important component of testing for the Altair lunar lander, an integral part of NASA's Constellation Program. It will be filled with liquid methane and extensively tested in a simulated lunar thermal environment to determine how liquid methane would react to being stored on the moon.

The volume of a sphere is a mathematical quantity that can be extended to spaces with different numbers of dimensions with some very interesting, and surprising, consequences!

The mathematical formula for the volume of a sphere in a space of N dimensions is given by the recursion relation

$$V(N) = \frac{2\pi R^2}{N} V(N-2)$$

For example, for 3-dimensional space, N = 3 and since from the table to the left, V(N-2) = V(1) = 2R, we have the usual formula

$$V(3) = \frac{4}{3}\pi R^3$$

Dimension	Formula	Volume
0	1	1.00
1	2R	2.00
2	πR^2	3.14
3	$\frac{4}{3}\pi R^3$	4.19
4		
5		
6		
7		
8		
9		
10		

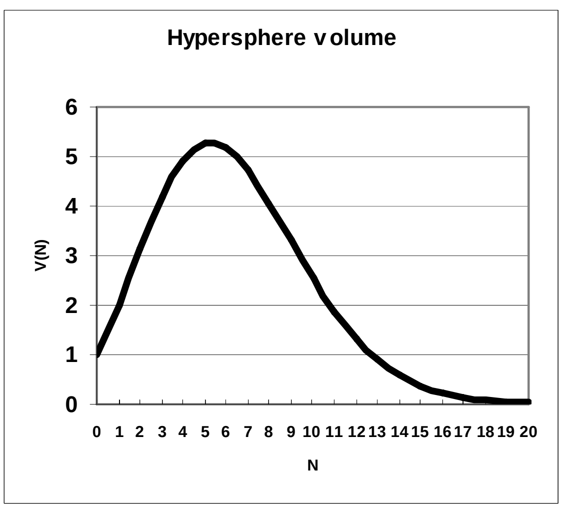
Problem 1 - Calculate the volume formula for 'hyper-spheres' of dimension 4 through 10 and fill-in the second column in the table.

Problem 2 - Evaluate each formula for the volume of a sphere with a radius of 1.00 and enter the answer in column 3.

Problem 3 - Create a graph that shows V(N) versus N. For what dimension of space, N, is the volume of a hypersphere its maximum possible value?

Problem 4 - As N increases without limit, what is the end behavior of the volume of an N-dimensional hypersphere?

Dimension	Formula	Volume
0	1	1.00
1	2R	2.00
2	πR^2	3.14
3	$\frac{4}{3}\pi R^3$	4.19
4	$\frac{\pi^2 R^4}{2}$	4.93
5	$\frac{8\pi^2 R^5}{15}$	5.26
6	$\frac{\pi^3 R^6}{6}$	5.16
7	$\frac{16\pi^3 R^7}{105}$	4.72
8	$\frac{\pi^4 R^8}{24}$	4.06
9	$\frac{32\pi^4 R^9}{945}$	3.30
10	$\frac{\pi^5 R^{10}}{120}$	2.55



Problem 1 - Answer for N=4:

$$V(4) = \frac{2\pi R^2}{4} V(4-2)$$

$$V(4) = \frac{2\pi R^2}{4} V(2)$$

$$V(4) = \frac{2\pi R^2}{4} (\pi R^2)$$

$$V(4) = \frac{\pi^2 R^4}{2}$$

Problem 2 - Answer for N=4:
$V(4) = (0.5)(3.141)^2 = 4.93$.

Problem 3 - The graph to the left shows that the maximum hypersphere volume occurs for spheres in the fifth dimension (N=5). Additional points have been calculated for N=11-20 to better illustrate the trend.

Problem 4 - In the limit for spaces with very large dimensions, the hypersphere volume approaches zero!