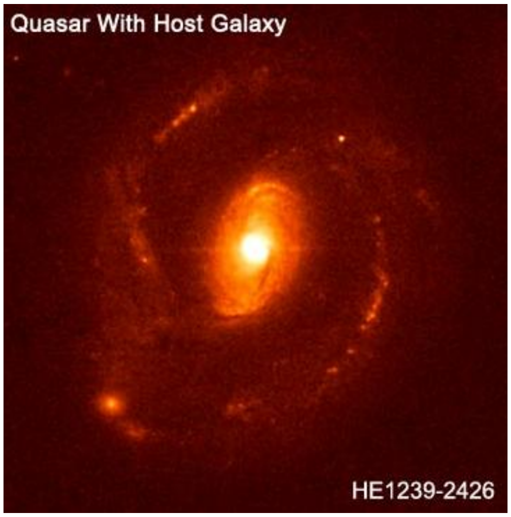
How Many Quasars Are There?



This Hubble Space Telescope image shows the brilliant 'quasar' core of a distant galaxy located 1.5 billion light years from Earth in the direction of the constellation Corvus the Crow.

Quasars are among the most luminous galaxies in the universe, and because of this, astronomers can detect them to great distances exceeding nearly 10 billion light years from Earth. Originally discovered as peculiar faint 'blue stars' in 1964, astronomers have counted up their numbers as a function of their brightness, expressed according to the logarithmic stellar apparent magnitude scale, m. The function that provides a good match to the quasar surface density, N(m), defined over the domain [+13.0, +23.0], is given by:

$$\frac{dN(m)}{dm} = 10^{f(m)}$$

where f(m) is a piecewise function defined as follows:

f(m) = +0.7m -12.0 for $+13.0 \leq m \leq +18.0$
f(m) = +0.5m -9.0 for $+19.0 \leq m \leq +23.0$

Problem 1 - The integral of dN(m)/dm is the total number of quasars present in one square degree of the sky. As a comparison, the full moon subtends an area of 1/4 square degree. How many quasars does the function predict that have magnitudes between +13.0 and +18.0 inclusively, over an area of the sky similar to five times the area of the full moon?

Problem 2 - How many quasars does the function predict that have magnitudes between +19.0 and +23.0 inclusively, over an area of the sky similar to five times the area of the full moon?

Problem 3 - An astronomer wants to study quasars in the brightness interval +13.0 < m < +18.0. What is the minimum sky area, in square degrees, that she needs to photograph in order to have one quasar present in the photograph?

Problem 4 - Combining the quasar sky densities from Problem 1 and 2, and the fact that the sky has a total angular area of 41,253 deg^2 ,how many quasars are there with magnitudes in the range from +13 to +23, inclusively?

Problem 1 - The integral of dN(m)/dm is the total number of quasars present in one square degree of the sky. As a comparison, the full moon subtends an area of 1/4 square degree. How many quasars does the function predict that have magnitudes between +13.0 and +18.0 inclusively, over an area of the sky similar to five times the area of the full moon?

Answer: $N = \int_{13}^{18} \frac{dN(m)}{dm} dm$ so from the definition of f(m) in the appropriate interval,

we get $N = \int_{13}^{18} 10^{0.7m-12} dm$ With u = (0.7m-12) , du = 0.7 dm, and using $10^u = e^{2.3u}$ the

equivalent integral becomes: $N = \frac{1}{0.7}\int_{-2.9}^{0.6} e^{2.3u} du$ or $N = \frac{1}{0.7(2.3)}\int_{-1.26}^{0.26} e^{x} dx$

so N = 0.62 [$e^{0.26}$ - $e^{-1.26}$] = 0.62[1.29 - 0.28] = 0.63 quasars per deg^2. The full moon has an area of 1/4 square degrees, so there will be 5 x (1/4 deg^2) x (0.63 quasars/deg^2) = **0.78 quasars in this sky area.**

Problem 2 - How many quasars does the function predict that have magnitudes between +19.0 and +23.0 inclusively, over an area of the sky similar to five times the area of the full moon?

Answer; $N = \int_{19}^{23} 10^{0.5m-9} dm$ With u = (0.5m-9) , du = 0.5 dm, and using $10^u = e^{2.3u}$

the equivalent integral becomes: $N = \frac{1}{0.5(2.3)}\int_{0.22}^{1.1} e^{x} dx$

so N = 0.87 [$e^{1.1}$ - $e^{0.22}$] = 0.87 [3.00 - 1.24] = 1.53 quasars/deg^2. For an area equal to 5 times the full moon, there are 5 x (1/4 deg^2) x (1.53 quasars/deg^2) = **1.9 quasars in this sky area.**

Problem 3 - An astronomer wants to study quasars in the brightness interval +13.0 < m < +18.0. What is the minimum sky area, in square degrees, that she needs to photograph in order to have one quasar present in the photograph? Answer; From Problem 1, the integral over this magnitude range gives an surface area of 0.63 quasars per square degree. To find one quasar, you need to photograph an area of 1/0.63 square degrees or **1.6 square degrees**. Note: The full moon occupies an area of 0.25 square degrees, so the astronomer will have to photograph an area of the sky about six times the area of the full moon.

Problem 4 - Answer: The combined quasar density is 0.63 + 1.53 = 1.16 quasars/deg^2 , so over the entire sky, there are about 1.16 x 41,253 = **47,853 quasars** in this magnitude range.