Exploring the Night-time Launch of STEREO

This spectacular night-time image taken by photographer Dominic Agostini shows the launch of the STEREO mission from Pad 17B at the Kennedy Space Center. The duration of the time-lapse image was 2.5 minutes. The distance to the horizon was 10 km, and the width of the image was 40 degrees. Assume that the trajectory is at the distance of the horizon, and in the plane of the photograph (the camera was exactly perpendicular to the plane of the launch trajectory).

Problem 1 – By using trigonometry, what is the horizontal scale of this image in meters/mm at the distance of the horizon?

Problem 2 - What was the altitude, in kilometers, of the rocket when it crossed the point directly in front of the camera's field of view at a position 2.4 km from the launch pad?

Problem 3 – From three measured points along the trajectory, what is the formula for the simplest parabola that can be fitted to this portion of the rocket's trajectory with the form $H(x) = ax^2 + bx + c$? (use kilometers for all measurements)

Problem 4 – Because of projection effects, although the rocket is continuing to gain altitude beyond the vertex of the parabola shown in the photograph, it appears as though the rocket reaches a maximum altitude and then starts to lose altitude as it travels further from the launch pad. About what is the apparent maximum altitude, in kilometers, that the rocket attains from the vantage point of the photographer?

More of Dominic's excellent night launch images can be found at www.dominicphoto.com

Problem 1 – By using trigonometry, what is the horizontal scale of this image in meters/mm at the distance of the horizon?
Answer: Solve the right-triangle to determine the hypotenuse h as h = 10 km/cos(20) = 11 km, then the width of the field of view at the horizon is just 2(11sin(20)) = 2(3.8 km) = 7.6 km. This is the width of the photograph along the distant horizon. With a millimeter ruler, the width of the mage is 125 mm, so the scale of this image at the horizon is about 7600m/125mm = **61 meters/mm**.

Problem 2 - What was the altitude, in kilometers, of the rocket when it crossed the point directly in front of the camera's field of view at a position 2.4 km from the launchpad? Answer: Draw a vertical line through the center of the image. Measure the length of this line between the distant horizon and the point on the trajectory. Typical values should be about 40 mm. Then from the scale of the image of 61 meters/mm, we get an altitude of H = 40 mm x 61 m/mm = **2440 meters or 2.4 kilometers**.

Problem 3 – From three measured points along the trajectory, what is the formula for the simplest parabola that can be fitted to this trajectory with the form $H(x) = ax^2 + bx + c$? (use kilometers for all measurements with the launch pad as the origin of the coordinates)

Answer: Because the launch pad is the origin of the coordinates, one of the two x-intercepts must be at (0,0) so the value for c = 0. We already know from Problem 2 that a second point is (2.4,2.4) so that $2.4 = a(2.4)^2 + b(2.4)$ and so the first equation for the solution is just
1 = 2.4a + b. We only need one additional point to solve for a and b. If we select a point at (1.2,1.5) we have a second equation $1.5 = a(1.2)^2 + 1.2b$ so that 1.5 = 1.4a + 1.2b. Our two equations to solve are now just

2.4 a + 1.0 b = 1.0
1.4 a + 1.2 b = 1.5

which we can solve by elimination of b to get a = -0.2 and then b = 1.48
so $\mathbf{H(x) = -0.2\, x^2 + 1.48\, x}$

Problem 4 – Because of projection effects, although the rocket is continuing to gain altitude beyond the vertex of the parabola shown in the photograph, it appears as though the rocket reaches a maximum altitude and then starts to lose altitude as it travels further from the launch pad. About what is the apparent maximum altitude, in kilometers, that the rocket attains from the vantage point of the photographer?

Answer: Students can use a ruler to determine this as about **2.7 kilometers**. Alternatively they can use the formula they derived in Problem 3 to get the vertex coordinates at
x = -b/2a
= +3.7 km and so h = **2.7 km**.