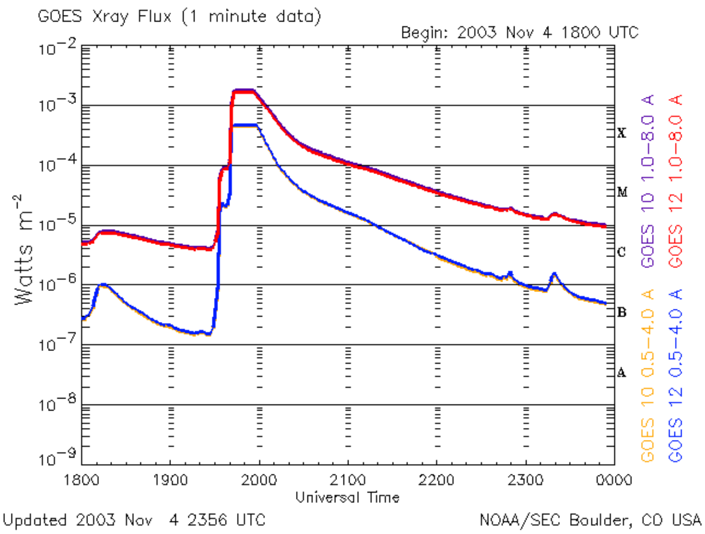
Solar Flare Reconstruction



During the November 4, 2003 solar flare, the GOES satellite measured the intensity of the flare as its light increased to a maximum and then decreased. The problem is that the solar flare was so bright that it could not record the most intense phase of the brightness evolution - what astronomers call its light curve. The figure above shows the light curve for two different x-ray energies, and you can see how its most intense phase near 19:50 UT has been clipped. This is a common problem with satellite detectors and is called 'saturation'. To work around this problem to recover at least some information about the flare's peak intensity, scientists resorted to mathematically fitting the pieces of the light curve that they were able to measure, and interpolated the data using their mathematical model, to estimate the peak intensity of the flare.

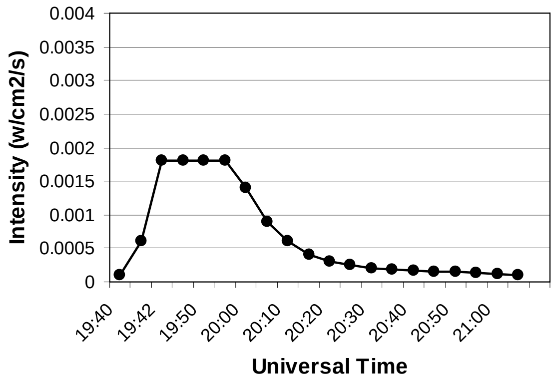
X-ray Flare Data.

Universal Time (UT)	Intensity (Watts/m^2/sec)
19:40	0.00010
19:41	0.00060
19:42	0.00180
19:45	saturated
19:50	saturated
19:55	0.00180
20:00	0.00140
20:05	0.00090
20:10	0.00060
20:15	0.00040
20:20	0.00030
20:25	0.00025
20:30	0.00020
20:35	0.00019
20:40	0.00017
20:45	0.00016
20:50	0.00015
20:55	0.00014
21:00	0.00012
21:05	0.00010

Problem 1 (Pre-Algebra): Re-plot the data in the table and from the trend on either side of the saturated region, estimate the peak intensity.

Problem 2 (Algebra): Re-plot the data, and from the information on either side of the saturation region, create two exponential functions that fit the data. Use the elapsed time since 19:40 as the independent variable. Find the intersections of these two functions to estimate the peak intensity and time.

Problem 3 (Calculus): Integrate the piecewise function in Problem 2 to determine the area under the light curve to 21:05. Note: 1 Watt equals 1 Joule of energy per second. Given that the sun is 147 million kilometers from the GOES satellite, calculate the surface area, in square meters, of a sphere of this radius. Calculate the total energy, in Joules, radiated by the flare that passed through the surface area of the sphere.



Re-plotted data to left, allowing extra space for interpolation.

Problem 1 (Pre-Algebra):

Answer: The curves, drawn free-hand, intersect between 0.0035 to 0.004 Watts/m^2/sec

Problem 2 (Algebra): Create two exponential functions that fit the data. Use the elapsed time since 19:40 as the independent variable.

Rising: From (0.0, 0.0001), (1.0, 0.0006) and (2.0, 0.00018) a best-fit exponential curve is $R(T) = 0.0001\ e^{(+1.44T)}$

Falling: From (20.0, 0.0018), (25.0, 0.0014), (30.0, 0.0009) and (35.0, 0.0006) a best-fit exponential curve is $R(T) = 0.0074\ e^{(-0.07T)}$

Find the intersections of these two functions to estimate the peak intensity and time.

$0.0001\ e^{1.44T} = 0.0074\ e^{-0.07T}$

Taking $\log_e$ of both sides : $\log_e(0.0001) + 1.44\ T = \log_e(0.0074) - 0.07T$

solve for T to get: $T = (+9.21 - 4.91)/1.51 = 2.84$ minutes

So the peak UT is 19:40 + 2.84 = 19:42.84 or 19:42:50

The peak intensity is then $0.0001\ e^{(1.44 \times 2.84)} = 0.006$ Watts/m^2

Problem 3 (Calculus): Integrate the piecewise function in Problem 2 to determine the area under the light curve. Note 1 watt x 1 second = 1 Joule.

Rising-side: From 0 to 2.84 minutes: $0.0001 \times (1/1.44)\ [\ e\ (1.44 \times 2.84) - 1\ ] = 0.0041$ Joules/m^2

Falling side from 2.84 to 85 minutes:

$0.0074 \times (1/0.07)\ [\ e^{(-0.070 \times 2.84)} - e^{(-0.070*85.0)}]$

$= 0.106\ [0.820 - 0.0026]$

$= 0.0867$ Joules/m^2

Combining we get a total 'flux' of 0.091 Joules/m^2

Given that the sun is 147 million kilometers from the GOES satellite, calculate the surface area of a sphere of this radius. Calculate the total energy radiated by the flare in ergs that passed through the surface area of the sphere.

Area = $4\pi(147 \times 10^6\ km)^2 = 2.71 \times 10^{17}\ km^2 \times 1.0 \times 10^6\ meter^2/km^2 = 2.71 \times 10^{23}\ m^2$

Total energy = 0.091 Joules/$m^2 \times 2.71 \times 10^{23}\ m^2 = 2.5 \times 10^{22}$ Joules