Black Holes...Hot Stuff!



An artist's impression of a black hole orbiting a companion star, and gravitationally attracting gas from the star into an orbiting accretion disk. Through friction, the gas becomes hotter as it approaches the black hole, turning from red to yellow to white. Most of the gas is swallowed by the black hole, but some is magnetically launched in jets away from the black hole at almost the speed of light. (Credit: M. Weiss, NASA/Chandra) The farther a particle falls towards a black hole, the faster it travels, and the more kinetic energy it has. Kinetic energy is mathematically defined as K.E. = $1/2 \text{ m V}^2$ where m is the mass of the particle and V is its speed.

Suppose all this energy is converted into heat energy by friction as the particle falls, and that this added energy causes nearby gases to heat up. How hot will the gas get? The equivalent amount of thermal energy, T.E., carried by a single particle is

$$T.E. = \frac{3}{2}kT$$

where Boltzman's Constant $k = 1.38 \times 10^{-23}$ Joules/deg. If we set K.E = T.E we get

$$T = \frac{mV^2}{3k}$$

If all the particles in a gas carried this same kinetic energy, then we would say the gas has a temperature of T in kelvins. We also know that the potential energy of the particle is given by

$$P.E. = \frac{GMm}{R}$$

So if we set P.E = T.E we also get the temperature

$$T = \frac{2GMm}{3kR}$$

Problem 1 - The formula T = 2 G M m/(3kR) gives the approximate temperature of hydrogen gas (m = 1.6×10^{-27} kg) in an accretion disk around a black hole. To two significant figures, what is the temperature for the material at the distance of Earth's orbit for a solar-mass black hole? (R = 1.47×10^{11} m, M = 1.9×10^{30} kg, for the constant of gravity G = 6.67×10^{-11} Nt m²/kg²)?

Problem 2 - How hot would the disk be at the distance of Neptune ($R = 4.4 \times 10^{12}$ meters)?

Problem 3 - X-rays are the most common forms of energy produced at temperatures above 100,000 K. Visible light is produced at temperatures above 2,000 K. Infrared radiation is commonly produced for temperatures below 500 K. What would you expect to see if you studied the accretion disk around a solar-mass sized black hole?

Answer Key:

Problem 1 - The formula T = 2/3 G M m/kR gives the approximate temperature of hydrogen gas (m = 1.6×10^{-27} kg) in an accretion disk around a black hole. To two significant figures, what is the temperature for a solar-mass black hole disk near the orbit of Earth? (R = 1.47×10^{11} m, M = 1.9×10^{30} kg, for G = 6.67×10^{-11} Nt m²/kg²)?

Answer: $T = 2/3 \times 6.67 \times 10^{-11} \times 1.9 \times 10^{30} \times 1.6 \times 10^{-27} / (1.38 \times 10^{-23} \times 1.47 \times 10^{11})$ = 65,000 K. to 2 significant figures

Problem 2 - How hot would the disk be at the distance of Neptune ($R = 4.4 \times 10^{12}$ meters)?

Answer:
$$T = 2/3 \times 6.67 \times 10^{-11} \times 1.9 \times 10^{30} \times 1.6 \times 10^{-27} / (1.38 \times 10^{-23} \times 4.4 \times 10^{12})$$

= 2,200 K.

Problem 3 - X-rays are the most common forms of energy produced at temperatures above 100,000 K. Visible light is produced at temperatures above 2,000 K. What would you expect to see if you studied the accretion disk around a solar-mass sized black hole?

Answer: The inner disk region would be an intense source of x-rays and visible light, because the gas is mostly at temperatures above 65,000 K. In the outer disk, the gas is much cooler and emits mostly visible or infrared light.