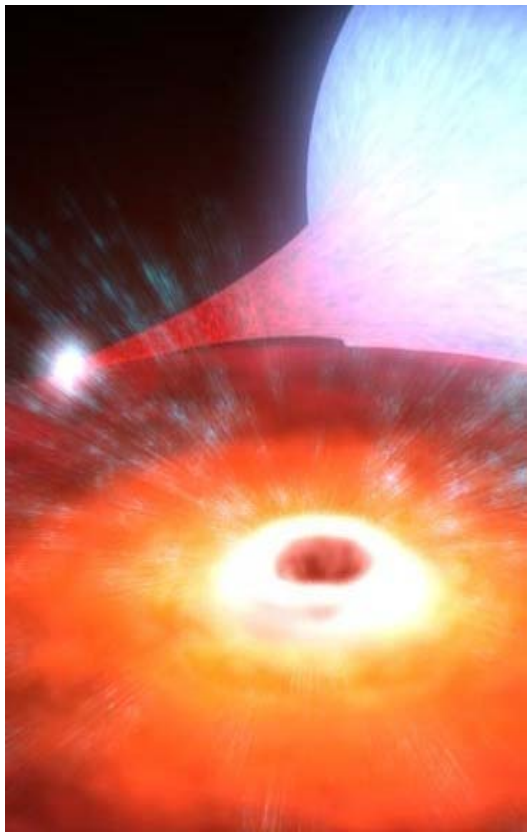
Falling Into a Black Hole



An object that falls into a black hole will cross the Event Horizon, and speed up as it gets closer. This is like a ball traveling faster and faster as it is dropped from a tall building. Suppose the particle fell 'from infinity', how fast would it be traveling? We can answer this question by considering the concepts of kinetic energy (K.E.) and gravitational potential energy (P.E.):

$$K.E. = 1/2\ m\ V^2$$

$$P.E. = G\ M\ m\ /\ R$$

The kinetic energy that the particle with mass, m, will gain as it falls, will depend on the total potential energy it has lost in traveling from infinity to a distance R. By setting the two equations equal to each other, we can relate the kinetic energy a particle gains as it falls to its current distance of R from the center of mass. The quantity, M, is the mass of the gravitating body the particle is falling towards. G is the constant of gravitation which equals 6.67×10^{-8} dynes cm^2/gm^2

$$1/2\ m\ V^2 = G\ M\ m\ /\ R$$

We can then solve for the speed, V, in terms of R

$$V = (\ 2GM\ /\ R\ )^{1/2}$$

Problem 1 - Suppose a body falls to Earth and strikes the ground. How fast will it be traveling when it hits if $M = 5.9 \times 10^{24}$ kilograms and R = 6,378 kilometers? Explain why this is the same as Earth's escape velocity?

Problem 2 - NASA's ROSSI satellite was used in 2004 to determine the mass and radius of a neutron star in the binary star system named EXO 0748-676, located about 30,000 light-years away in the southern sky constellation Volans, or Flying Fish. The neutron star was deduced to have a mass of 1.8 times the sun, and a radius of 11.5 kilometers. A) How fast, in km/sec, will a particle strike the surface of the neutron star if the mass of the sun is 1.9×10^{33} grams? B) In percent, what will the speed be compared to the speed of light, 300,000 km/sec?

Problem 3 - The star HD226868 is a binary star with an unseen companion. It is also the most powerful source of X-rays in the sky second to the sun - it's called Cygnus X-1. Astronomers have determined the mass of this companion to be 8.7 times the sun. As a black hole, its Event Horizon radius would be $R = 2.95 \times 8.7 = 26$ kilometers. A) How fast, in km/sec, would a body be traveling as it passed through the Event Horizon? B) In percentage compared to the speed of light?

Answer Key:

Problem 1 - Suppose a body falls to Earth and strikes the ground .How fast will it be traveling when it hits? Explain why this is the same as Earth's escape velocity?

Answer - R = 6,378 kilometers. M = 5.9 x 10^{24} kilograms.
V = (2 x 6.67 x 10^{-8} x 5.9 x 10^{29}/6.4 x 10^{8})$^{1/2}$ = 1.1 x 10^{6} cm/sec or 11 kilometers/second. The particle 'fell' from infinity, so this means that if you gave a body a speed of 11 km/sec at Earth's surface, it would be able to travel to infinity and 'escape' from Earth.

Problem 2 - NASA's ROSSi satellite was used in 2004 to determine the mass and radius of a neutron star in the binary star system named EXO 0748-676, located about 30,000 light-years away in the southern sky constellation Volans, or Flying Fish. The neutron star was deduced to have a mass of 1.8 times the sun, and a radius of 11.5 kilometers. A) How fast, in km/sec, will a particle strike the surface of the neutron star if the mass of the sun is 1.9 x 10^{33} grams? B) In percent, what will the speed be compared to the speed of light, 300,000 km/sec?

Answer: A) Mass = 1.8 x 1.9 x 10^{33} grams = 3.4 x 10^{33} grams. V = (2 x 6.67x10^{-8} x 3.4 x 10^{33}/1.15 x 10^{6})$^{1/2}$ = 1.98 x 10^{10} cm/sec = 198,000 km/sec.

B) 198,000/300,000 = 66 % of the speed of light!

Problem 3 - The star HD226868 is a binary star with an unseen companion. It is also the most powerful source of X-rays in the sky second to the sun - it's called Cygnus X-1. Astronomers have determined the mass of this companion to be 8.7 times the sun. As a black hole, its Event Horizon radius would be R = 2.95 km x 8.7 = 26 kilometers. A) How fast, in km/sec, would a body be traveling as it passed through the Event Horizon? B) In percentage compared to the speed of light?

Answer: A) Mass = 8.7 x 1.9 x 10^{33} grams = 1.7 x 10^{34} grams. V = (2 x 6.67x10^{-8} x 1.7 x 10^{34}/2.55 x 10^{6})$^{1/2}$ = 2.98 x 10^{10} cm/sec = 298,000 km/sec.

B) 298,000/300,000 = 99.3% of the speed of light!