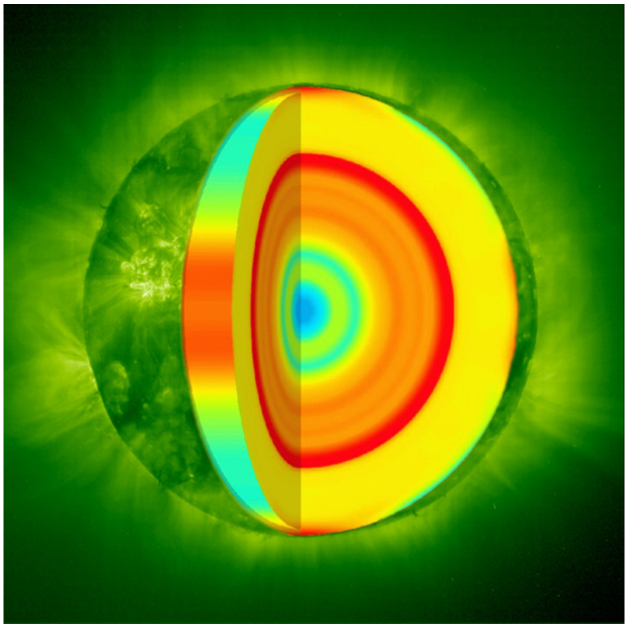
A Mathematical Model of the Sun



Once an astronomer knows the radius and mass of a planet or star, one of the first things they might do is to try to figure out what the inside looks like.

Gravity would tend to compress a large body so that its deep interior was at a higher density than its surface layers. Imagine that the sun consisted of two regions, one of high density (the core shown in red) and a second of low density (the outer layers shown in yellow). All we know about the sun is its radius (696,000 km) and its mass (2.0 x 10^{30} kilograms). What we would like to estimate is how dense these two regions would be, in order for the final object to have the volume of the sun and its total observed mass. We can use the formula for the volume of a sphere, and the relationship between density, volume and mass to create such a model.

Suppose the inner zone has a radius of 417,000 km. The volume of this 'core' is

$$V_c = \frac{4}{3} \pi \,(417{,}000{,}000 \text{ m})^3 = 3.0 \times 10^{26} \text{ m}^3$$

The volume of the entire sun is

$$V^* = \frac{4}{3} \pi \,(696{,}000{,}000 \text{ m})^3 = 1.4 \times 10^{27} \text{ m}^3$$

So the outer shell zone has a volume of

$$V_s = V^* - V_c$$

So, $V_s = 1.1 \times 10^{27} \text{ km}^3$

Mass = Density x Volume,

so if we estimate the density of the sun's core as Dc = 100,000 kg/m^3, and the outer shell as Ds = 10,000 kg/m^3, the masses of the two parts are:

$M_c = 100{,}000 \times (3.0 \times 10^{26} \text{ m}^3) = 3.0 \times 10^{31}$ kg

$M_s = 10{,}000 \times (1.1 \times 10^{27} \text{ m}^3) = 1.1 \times 10^{31}$ kg

So the total solar mass in our model would be:

$$M^* = M_c + M_s = 4.1 \times 10^{31} \text{ kg}$$

This is about 20 times more massive than the sun actually is!

Inquiry Problem.

With the help of an Excel spreadsheet, program the spreadsheet so that you can adjust the radius of the core and calculate the volume of the core and shell zones. Then create a formula so that you can enter various choices for the densities of the core and shell zones and then sum-up the total mass of your mathematical model for the sun.

What are some possible ranges for the core radius, and zone densities (in grams per cubic centimeter) that give about the right mass for the modeled sun? How do these compare with what you find from searching the web literature? How could you improve your mathematical model to make it more accurate? Show how you would apply this method to modeling the interior of the Earth, or the planet Jupiter?

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Notes: The big challenge for students who haven't used the volume formula is how to work with it for large objects…requiring the use of scientific notation as well as learning how to use the volume formula. Students will program a spreadsheet in whatever way works best for them, although it is very helpful to lay out the page in an orderly manner with appropriate column labels. Students, for example, can select ranges for the quantities, and then generate several hundred models by just copying the formula into several hundred rows. Those familiar with spreadsheets will know how to do this ,and it saves entering each number by hand…which would just as easily have been done with a calculator and does not take advantage of spreadsheet techniques.

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Notes: Most solar models suggest a core density near 160 grams/cc, and an outer 'convection zone' density of about 0.1 grams/cc. The average value for the shell radius is about 0.7 x the solar radius. Students will encounter, using GOOGLE, many pages on the solar interior, but the best key words are things like 'solar core density' and 'convection zone density'. These kinds of interior models are best improver by 1) using more shells to divide the interior of the body, and 2) developing a mathematical model of how the density should change from zone to zone. Does the density increase linearly as you go from the surface to the core, or does it follow some other mathematical function? Students may elect to also test various models for the interior density change such as

$$D = \frac{100 \text{ gm/cc}}{R} \quad \text{or even} \quad 100 \text{ gm/cc} \times e^{-R/R^*}$$

This method can be applied to any spherical body (planet, asteroid, etc) for which you know a radius and mass ahead of time.

Alternately, if you know a planet's mass and its composition (this gives an average density), then you can estimate its radius!