Equation 1

$$V = \sqrt{2\,g\,H}$$

Equation 2

$$X = \frac{V^2}{g}$$

Equation 3

$$T = \frac{V\sqrt{2}}{g}$$

There are three equations that describe projectile motion on a planet:

Equation 1: Maximum velocity, **V**, needed to reach a height, **H**:

Equation 2: Maximum horizontal distance, **X**:

Equation 3: Time, **T**, required to reach maximum horizontal distance:

In all three equations, **g** is a constant and is the acceleration of gravity at the surface of the planet, and all units are in meters or seconds.

Problem 1 - The volcano, Krakatoa, exploded on August 26, 1883 and obliterated an entire island. The detonation was heard over 2000 kilometers away in Australia, and was the loudest sound created by Nature in recorded human history! If the plume of gas and rock reached an altitude of H=17 miles (26 kilometers) what was the speed of the gas, V, that was ejected, in A) kilometers/hour? B) miles/hour? C) What was farthest horizontal distance, X, in kilometers that the ejecta reached? D) How long, T, did it take for the ejecta to travel the maximum horizontal distance? E) About 30,000 people were killed in the explosion. Why do you think there were there so many casualties? (Note: **g** = 9.8 meters/sec^2 for Earth.)

Problem 2 - An asteroid collides with the lunar surface and ejects lunar material at a speed of V=3,200 kilometers/hr. A) How high up, H, does it travel before falling back to the surface? B) The escape speed from the lunar surface is 8,500 km/hr. From your answer to Problem 1, would a 'Krakatoa' explosion on the moon's surface have been able to launch lunar rock into orbit? (Note: **g** = 1.6 meters/sec^2 for the Moon.)

Problem 3 - Plumes of gas are ejected by geysers on the surface of the satellite of Saturn called Enceladus. If **g** = 0.1 meters/sec^2 , and H = 750 km, what is the speed of the gas, V, in the ejection in kilometers/hr?

Inquiry Problem: Program an Excel Spreadsheet to calculate the various quantities in the three equations given input data about the planet and ejecta. How does the maximum ejection velocity and height change with the value of **g** used for a variety of bodies in the solar system?

Answer Key

Problem 1 - The volcano, Krakatoa, exploded on August 26, 1883 and obliterated an entire island. The detonation was heard over 2000 kilometers away in Australia, and was the loudest sound created by Nature in recorded human history! If the plume of gas and rock reached an altitude of 17 miles (26 kilometers) what was the speed of the gas that was ejected, in

A) Use Equation 1 with H = 26,000 meters; g = 9.8 m/s^2 and get H = (2 x 26000x9.8)$^{1/2}$ = 714 m/sec, but since the input numbers are only good to two significant figures, the answer is 710 meters/sec. Then converting to km/hr we get 710 m/s x (3600 sec/hr) x(1 km/1000meters) = 2,556 km/hr but again we only report to 2 significant figures so the answer is **2,600 km/hr**.

B) 2,600 km/hr x (0.62 miles/km) = **1,600 miles/hour** to 2 significant figures

C) Use Equation 2: X = (710 meters/sec)^2/9.8 = 51,439 meters, which to 2 significant figures becomes 51,000 meters or **51 kilometers**.

D) Use Equation 3: T = 1.414 x (710)/9.8 = 102.4, but to 2 significant figures is **100 seconds.**

E) About 30,000 people were killed in the explosion. Why do you think there were there so many casualties? Answer: **They had less than 100 seconds to flee from the advancing ejecta cloud!** You can also ask the students to calculate the sound travel time to cross 51 kilometers (sound speed = 340 m/sec) which would take 51,000 meters/340 = 150 seconds to reach someone at 51 kilometers…so the eject would strike them BEFORE they even heard the detonation.

Problem 2 - Answer: 3,200 km/hr = 0.9 km/sec = **900 meters/sec**. to 2 significant figures. A) Solve Equation 1 for H… H = V2/2g so H = (900)^2/(2 x 1.6) = **250 kilometers** (2 SigFig). B) The escape speed from the lunar surface is 8,500 km/hr. From your answer to Problem 1, would a 'Krakatoa' explosion on the moon's surface have been able to launch lunar rock into orbit? (Note: g = 1.6 meters/sec^2 for the Moon.) Answer: **Yes!**

Problem 3 - Answer: From Equation V = (2 x 0.1 x 750000)$^{1/2}$ = 390 meters/sec = **1,400 km/hr** (2 SigFig)

Inquiry Problem: Program an Excel Spreadsheet to calculate the various quantities in the three equations given input data about the planet and ejecta. How does the maximum ejection velocity and height change with the value of **g** used for a variety of bodies in the solar system?

Answer: **There are many ways for students to program each column in a spreadsheet to calculate the variables in the equations. Students should, for instance, notice that as the surface gravity, g, increases, the maximum speed, V, changes as the square-root of g, and the values for X and T vary inversely with g.**