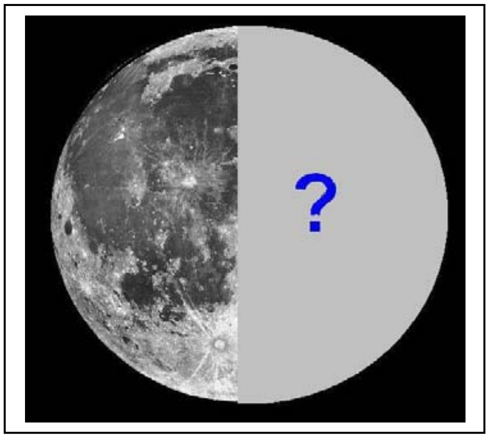
The Moon's Density - What's inside?



The Moon has a mass of 7.4×10^{22} kg and a radius of 1,737 km. Seismic data from the Apollo seismometers also shows that there is a boundary inside the Moon at a radius of about 400 km where the rock density or composition changes. Astronomers can use this information to create a model of the Moon's interior.

Problem 1 - What is the average density of the Moon in grams per cubic centimeter (g/cm^3) ? (Assume the Moon is a perfect sphere.)

Problem 2 - What is the volume, in cubic centimeters, of A) the Moon's interior out to a radius of 400 km? and B) The remaining volume out to the surface?

You can make a simple model of a planet's interior by thinking of it as an inner sphere (the core) with a radius of R(core), surrounded by a spherical shell (the mantle) that extends from R(core) to the planet's surface, R(surface). We know the total mass of the planet, and its radius, R(surface). The challenge is to come up with densities for the core and mantle and R(core) that give the total mass that is observed.

Problem 3 - From this information, what is the total mass of the planet model in terms of the densities of the two rock types (D1 and D2) and the radius of the core and mantle regions R(core) and R(surface)?

Problem 4 - The densities of various rock types are given in the table below.

Type	Density
I - Iron+Nickle mixture (Earth's core)	15.0 gm/cc
E - Earth's mantle rock (compressed)	4.5 gm/cc
B - Basalts	2.9 gm/cc
G - Granite	2.7 gm/cc
S - Sandstone	2.5 gm/cc

A) How many possible lunar models are there? B) List them using the code letters in the above table, C) If denser rocks are typically found deep inside a planet, which possibilities survive? D) Find combinations of the above rock types for the core and mantle regions of the lunar interior model, that give approximately the correct lunar mass of 7.4×10^{25} grams. [Hint: use an *Excel* spread sheet to make the calculations faster as you change the parameters.] E) If Apollo rock samples give an average surface density of 3.0 gm/cc, which models give the best estimates for the Moon's interior structure?

Problem 1 - Mass = 7.4×10^{22} kg x 1000 gm/kg = 7.4×10^{25} grams. Radius = 1,737 km x 100,000 cm/km = 1.737×10^{8} cm. Volume of a sphere = $4/3\ \pi\ R^3 = 4/3 \times (3.141) \times (1.737 \times 10^8)^3 = 2.2 \times 10^{25}\ cm^3$, so the density = 7.4×10^{25} grams / $2.2 \times 10^{25}\ cm^3$ = **3.4 gm / cm^3.**

Problem 2 - A) V(core) = $4/3\ \pi\ R^3 = 4/3 \times (3.141) \times (4.0 \times 10^7)^3$ **= 2.7×10^{23} cm^3**
B) V(shell) = V(Rsurface) - V(Rcore) = $2.2 \times 10^{25}\ cm^3 - 2.7 \times 10^{23}\ cm^3$ = **2.2×10^{25} cm^3**

Problem 3 - The total core mass is given by M(core) = $4/3\ \pi\ (Rcore)^3 \times D1$. The volume of the mantle shell is given by multiplying the shell volume V(shell) calculated in Problem 2B by the density : Mshell = V(shell) x D2. Then, the formula for the total mass of the model is given by: MT = $4/3\ \pi\ (Rc)^3 \times D1 + (4/3\ \pi\ (Rs)^3 - 4/3\ \pi\ (Rc)^3) \times D2$, which can be simplified to:

$$\mathbf{MT = 4/3\ \pi\ (D1 \times Rc^3 + D2 \times Rs^3 - D2 \times Rc^3)}$$

Problem 4 - A) There are 5 types of rock for 2 lunar regions so the number of unique models is 5 x 5 = 25 possible models. B) The possibilities are: II, IE, IB, IG, IS, EE, EI, EB, EG, ES, BI, BE, BB, BG, BS, GI ,GE, GB, GG, GS, SI, SE, SB, SG, SS. C) The ones that are physically reasonable are: IE, IB, IG, IS, EB, EG, ES, BG, BS, GS. The models, II, EE, BB, GG and SS are eliminated because the core must be denser than the mantle. D) Each possibility in your answer to Part C has to be evaluated by using the equation you derived in Problem 3. This can be done very efficiently by using an Excel spreadsheet. The possible answers are as follows:

Model Code	Mass (in units of 10^{25} grams)
I E	10.2
I B	6.7
E B	6.4
I G	6.3
E G	6.0
B G	6.0
I S	5.8
E S	5.5
B S	5.5
G S	5.5

E) The models that have rocks with a density near 3.0 gm/cc as the mantle top layer are the more consistent with the density of surface rocks, so these would be IB and EB which have mass estimates of 6.7×10^{25} and 6.4×10^{25} grams respectively. These are both very close to the actual moon mass of 7.4×10^{25} grams (e.g. 7.4×10^{25} grams) so it is likely that the moon has an outer mantle consisting of basaltic rock, similar to Earth's mantle rock (4.5 gm/cc) and a core consisting of a denser iron/nickel mixture (15 gm/cc).