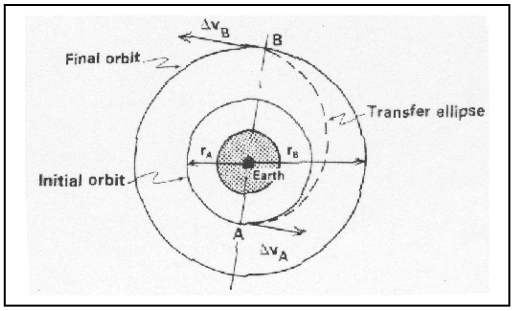
Fly me to the Moon!



If spacecraft had rockets that could make them travel at any speed, we could fly to the Moon from Earth in a straight line, and make the trip in a few minutes. In the Real World, we can't do that even with the most powerful rockets we have. Instead, we have to obey Newton's Laws of Motion and take more leisurely, round-about routes!

To see how this works, you need a compass, metric ruler, a large piece of paper, a string, a thumbtack, and a pencil.

Step 1 - With your compass, draw a circle 1/2-centimeter in radius. Label the inside of this 'Earth'. **Step 2** - Draw a second circle centered on Earth with a radius of 1 centimeter. Label this 'Earth Orbit'. **Step 3** - Using the string and thumbtack, draw a second circle with a radius of 30 centimeters. Label this 'Orbit of Moon'. **Step 4** - Draw a line connecting the center of earth and a point on the lunar orbit. Label the lunar orbit Point B. **Step 5** - Extend the line so that it intersects a point on the Earth orbit circle in the opposite direction from Earth's center. There should be two intersection points. The first will be between Earth and the lunar orbit. Label this Point C. The second will be behind Earth. Label this Point A. **Step 6** - As carefully as you can, draw a free-hand ellipse with one focus centered on Earth that arcs between Point A and Point B. This is called the major axis of the ellipse. See the above figure for comparison.

What you have drawn is a simple rocket trajectory, called a Hohmann Transfer orbit, that connects a spacecraft orbiting Earth, with a point on the lunar orbit path. If you had unlimited rocket energy, you could travel the path from Point C to Point B in a few hours or less. If you had less energy, you would need to take a path that looks more like the one from Point A to Point B and is slower, so it takes more time. Even less energy would involve a spiral path that connects Point A and Point B but may loop one or more times around Earth as it makes its way to lunar orbit. Can you draw such a path?

Problem 1 - If there were no gravity, spacecraft could just travel from place to place in a straight line at their highest speeds, like the Enterprise in *Star Trek*. If the distance to the Moon is 380,000 kilometers, and the top speed of the Space Shuttle is 10 kilometers/sec, how many hours would the Shuttle take to reach the Moon?

Astrodynamicists are the experts that calculate orbits for spacecraft. One of the most important factors is the total speed change, called the delta-V, to get from one orbit to another. For a rocket to get into Earth orbit requires a delta-V of 8600 m/sec. To go from Earth orbit to the Moon takes an additional delta-V of 4100 meters/sec.

Problem 2 - To enter a Lunar Transfer Orbit, a spacecraft has enough fuel to make a total speed change of 3500 m/sec. If it needs to make a speed change of 2000 m/s in the horizontal direction, and 3000 m/sec in the vertical direction to enter the correct orbit, is there enough fuel to reach the Moon in this way? [Hint, use the Pythagorean Theorem]

Answer Key

Problem 1 - If there were no gravity, spacecraft could just travel from place to place in a straight line at their highest speeds, like the Enterprise in *Star Trek*. If the distance to the Moon is 380,000 kilometers, and the top speed of the Space Shuttle is 10 kilometers/sec, how many hours would the Shuttle take to reach the Moon?

Answer - Distance = speed x time, so 380,000 km = 10 km/s x time. Solving for Time you get 38,000 seconds. Since there are 60 minute/hour x 60 seconds/minute = 3600 seconds/hour, 38,000 / 3600 = 10.6 hours.

Note: In reality, it takes several days for spacecraft to make the trip under the influence of gravity and allowing for conservation of energy and momentum.

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Answer:

<u>Method 1</u>: From the lengths of the horizontal and vertical speeds, we want to find the length of the hypotenuse of a right triangle. Using the Pythagorean Theorem

$$\text{total speed} = (2000^2 + 3000^2)^{1/2} = 3605 \text{ m/sec}$$

This is the total change of speed that is required, but there is only enough fuel for 3500 m/sec so the spacecraft cannot enter the Transfer Orbit.

<u>Method 2</u>: Graphically, draw a right triangle to the proper scale, for example, 1 centimeter = 1000 m/sec. Then the two sides of the triangle have lengths of 2 cm and 3 cm. Measure the length of the hypotenuse to get 3.6 cm, then convert this to a speed by multiplying by 1000 m/sec to get the answer.