Solar panels have to be installed carefully so that the tilt of the roof, and the direction to the sun, produce the largest possible electrical power in the solar panels.

A simple application of vector dot and cross products lets us predict the amount of electrical power the panels can produce.

A surveyor on the sidewalk uses his instruments to determine the coordinates of the four corners of a roof where solar panels are to be mounted. In the picture shown above, suppose the points are labeled counter clockwise from the roof corner nearest the camera in units of meters: P1(6, 8, 4); P2(21, 8, 4); P3(21, 16, 10) and P4(6, 16, 10)

Problem 1 – What are the components to the two edge vectors defined by **A** = **P**2-**P**1 and **B** = **P**4-**P**1. Write the vector in standard notation with **x**, **y** and **z** being the coordinate unit vectors.

Problem 2 – What are the magnitudes of the vectors **A** and **B,** and in what units?

Problem 3 – What are the components to the vector, **N**, perpendicular to **A** and **B** and the surface of the roof?

Problem 4 – What is the magnitude of **N** and its units?

Problem 5 – The sun is located along the unit vector **S** = ½ **x** – 6/7 **y** + 1/7**z**. If the flow of solar energy is given by the vector **F** = 910 **S** in units of watts/meter^2, what is the dot product of **F** with **N**, and the units for this quantity?

Problem 6 – What is the angle between **N** and **S**? What is the elevation angle of the sun above the plane of the roof?

Problem 1 – What are the components to the two edge vectors defined by **A** = P2-P1 and **B** = P4-P1. Write the vector in standard notation with **x**, **y** and **z** being the coordinate unit vectors.

Answer: **A** = (21-6)**x** +(8-8)**y** + (4-4)**z** so **A =** 15**x**
B = (6-6)**x** + (16-8)**y** + (10-4)**z** so **B =** 8**y +** 6**z**

Problem 2 – What are the magnitudes of the vectors **A** and **B**, and in what units?

Answer: ||**A**|| = **15 meters** ||**B**|| = $(8^2 + 6^2)^{1/2}$ = **10 meters**

Problem 3 – What are the components to the vector, **N**, perpendicular to **A** and **B** and the surface of the roof?

Answer: Use the vector cross product: **N** = **A** x **B** so **N** = 0**x** – 90**y** +120**z**

Problem 4 – What is the magnitude of **N** and its units?
Answer: ||**N**|| = $((-90)^2 + (120)^2)^{1/2}$ so ||**N**|| = 150 meters^2 which is the area of the roof

Problem 5 – The sun is located along the unit vector **S** = ½ **x** – 6/7 **y** + 1/7**z.** If the flow of solar energy is given by the vector **F** = 910 **S** in units of watts/meter^2, what is the dot product of **F** with **N**, and the units for this quantity?

Answer: **F** = 910 (1/2**x** -6/7**y** +1/7**z**) = 455**x** – 780**y** + 130**z**.

The dot product is just **F**dot**N** = 455*(0) -780*(-90) + 130*120 = **85,800 watts**.

Problem 6 – What is the angle between **N** and **S**? What is the elevation angle of the sun above the plane of the roof?

Answer: From the definition of dot product**:** **F** dot **N** = ||**F**|| ||**N**|| sin θ

Then since ||**F**|| = 910 and ||**N**|| = 150 and **F**dot**N** = 85,800 we have

Sinθ = (85800/(910x150)) = 0.629 and so θ **= 39°**. This is the angle between the normal to the surface and the incident solar rays. The compliment of this is the elevation of the sun above the plane of the roof or 90-39 = 51°.