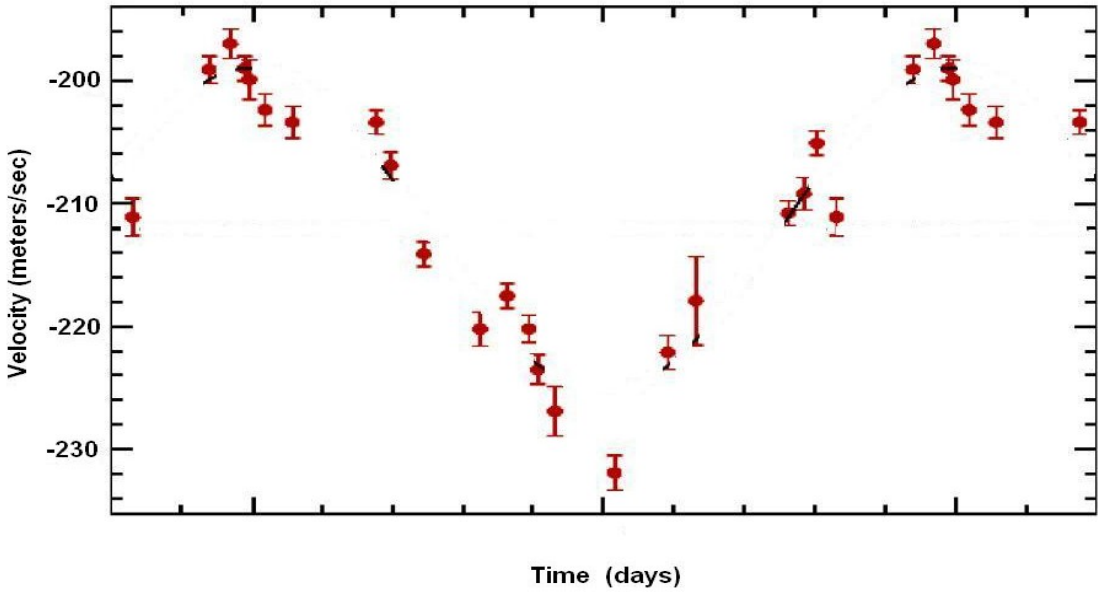
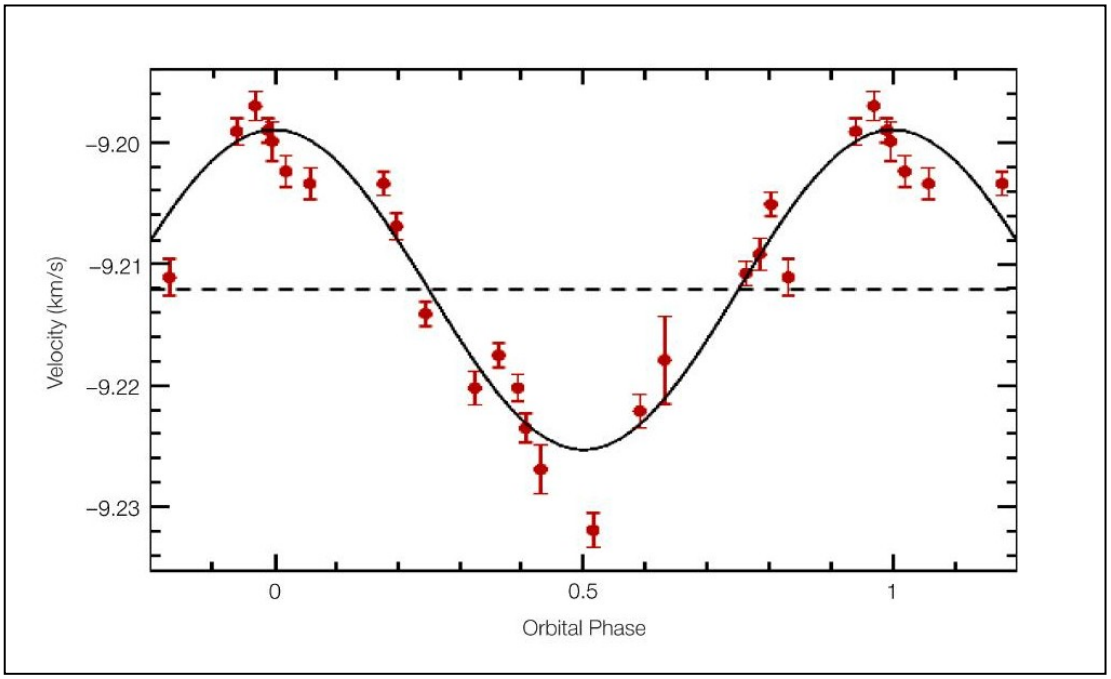
Fitting Periodic Functions - Distant Planets



A team of French and Swiss astronomers have discovered one of the lightest exoplanets ever found using the HARPS instrument on ESO's 3.6-m telescope at La Silla (Chile). They measured the speed of the star, Gleise-581 that the planet orbits, and plotted the data as shown above. The marks on the horizontal axis are spaced every 0.54 days apart starting at 0 which occurred on June 8, 2004.

Problem 1: From the data, create an estimate of the best-fit periodic function that follows the trend in the data. Calculate the amplitude, offset (vertical shift) phase and the formula for the angle in terms of the elapsed time in days since the start of the plot.

Problem 2: What would you predict as the velocity of the star on June 19, 2004?



The figure above shows the actual cosine fit proposed by the astronomers for the star's periodic motion. It is of the form V = amplitude x cos($\theta - \phi$) + v0

Problem 1: The two peaks occur at T = 2 x 0.54 = 1.08 days and T = 12 x 0.54 = 6.48 days. The first peak occurs at a velocity of -198 m/s. The first minimum occurs at -226 km/sec so the average value (dotted line) is v0 = –(198 + 226)/2 = **-212 km/sec.** The amplitude is found by taking the difference between the maximum and minimum and dividing by 2 to get (-198 – (-226))/2 = **14 m/sec.** The period between the peaks is 6.48 – 1.08 = 5.4 days, so the orbital period is 5.4 days during which time the angle goes through 2π radians. That means that for all other times, the angle will be $\theta = 2\pi$ T/5.4 or **0.37 π T** where T is the elapsed time in days since June 8, 2004. The first peak starts at 1.08 days, (where cos(0) = 1) so the phase-shift of the first peak from T=0 (June 8) is $\phi = 2\ \pi\ 1.08/5.4 =$ **0.4 π.** The best-fit equation is as follows:

$$\mathbf{V = 14 \cos(0.37\pi T - 0.4\ \pi) - 212\ km/s}$$

Problem 2: T = June 19-June 8 = 11 days, so (0.37 π 11 – 0.4 π) = 11.53 radians. Then V = 14 cos(11.53) – 212 **= -205 m/sec**

The scientific results were published in 2005, in the journal *Astronomy and Astrophysics*, vol 443, page L15. For more information, see the Press Release at **http://www.eso.org/public/outreach/press-rel/pr-2005/pr-30-05.html**