Using the Properties of Exponents

Astronomers rely on scientific notation in order to work with very large and small measurements. The rules for using this notation are pretty straight-forward, and are commonly taught in most 7th-grade math classes as part of the National Education Standards for Mathematics.

The following problems involve the conversion of decimal numbers into SN form, and are taken from common astronomical applications and quantities.

1) Length of a year. 31,560,000.0 seconds

2) Speed of light: 299,792.4 kilometers/sec

3) Mass of the sun: 1,989,000,000,000,000,000,000,000,000,000,000 grams

4) Mass of Earth: 5,974,000,000,000,000,000,000,000 kilograms

5) One light-year : 9,460,500,000,000 kilometers

6) Power output of sun : 382,700,000,000,000,000,000,000,000 watts

7) Mass of an electron: 0.00000000000000000000000000000091096 kilograms

8) Energy equivalent of one electron-Volt : 0.00000000000000000016022 joules

9) Ratio of proton to electron mass: 1,836.2

10) Planck's Constant: 0.000000000000000000000000006626068 ergs seconds

11) Radius of hydrogen atom : 0.00000000529177 centimeters

12) Radius of Earth's orbit: 14,959,789,200,000 centimeters

13) Smallest unit of physical distance: 0.0000000000000000000000000000000016 centimeters

14) Diameter of Visible Universe: 26,000,000,000,000,000,000,000,000,000 centimeters

Answer Key:

1) Length of a year. 31,560,000.0 seconds
Answer: 3.156×10^{7} seconds

2) Speed of light: 299,792.4 kilometers/sec
Answer: 2.997924×10^{5} km/sec

3) Mass of the sun: 1,989,000,000,000,000,000,000,000,000,000,000 grams
Answer: 1.989×10^{33} grams

4) Mass of Earth: 5,974,000,000,000,000,000,000,000 kilograms
Answer: 5.974×10^{24} kg

5) One light-year : 9,460,500,000,000 kilometers
Answer: 9.4605×10^{12} km

6) Power output of sun : 382,700,000,000,000,000,000,000,000 watts
Answer: 3.827×10^{26} watts

7) Mass of an electron: 0.00000000000000000000000000000091096 kilograms
Answer: 9.1096×10^{-31} kg

8) Energy equivalent of one electron-Volt : 0.00000000000000000016022 joules
Answer: 1.6022×10^{-19} Joules

9) Ratio of proton to electron mass: 1,836.2
Answer; 1.8362×10^{3}

10) Planck's Constant: 0.000000000000000000000000006626068 ergs seconds
Answer: 6.626068×10^{-27} ergs seconds

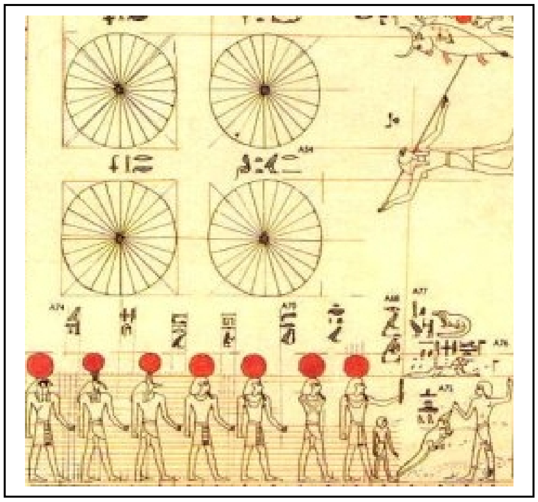
11) Radius of hydrogen atom : 0.00000000529177 centimeters
Answer: 5.29177×10^{-9} cm

12) Radius of Earth's orbit: 14,959,789,200,000 centimeters
Answer: $1.49597892 \times 10^{13}$ cm

13) Smallest unit of physical distance: 0.0000000000000000000000000000000016 centimeters
Answer: 1.6×10^{-33} cm

14) Diameter of Visible Universe: 26,000,000,000,000,000,000,000,000,000 centimeters
Answer; 2.6×10^{28} cm

Using Properties of Exponents



Astronomers rely on scientific notation in order to work with 'big' things in the universe. The rules for using this notation are pretty straight-forward, and are commonly taught in most 7th-grade math classes as part of the National Education Standards for Mathematics.

The following problems involve the addition and subtraction of numbers expressed in Scientific Notation. For example:

$$1.34 \times 10^{8} + 4.5 \times 10^{6} = 134.0 \times 10^{6} + 4.5 \times 10^{6}$$
$$= (134.0 + 4.5) \times 10^{6}$$
$$= 138.5 \times 10^{6}$$
$$= 1.385 \times 10^{8}$$

1) $1.34 \times 10^{14} + 1.3 \times 10^{12} =$

2) $9.7821 \times 10^{-17} + 3.14 \times 10^{-18} =$

3) $4.29754 \times 10^{3} + 1.34 \times 10^{2} =$

4) $7.523 \times 10^{25} - 6.32 \times 10^{22} + 1.34 \times 10^{24} =$

5) $6.5 \times 10^{-67} - 3.1 \times 10^{-65} =$

6) $3.872 \times 10^{11} - 2.874 \times 10^{13} =$

7) $8.713 \times 10^{-15} + 8.713 \times 10^{-17} =$

8) $1.245 \times 10^{2} - 5.1 \times 10^{-1} =$

9) $3.64567 \times 10^{137} - 4.305 \times 10^{135} + 1.856 \times 10^{136} =$

10) $1.765 \times 10^{4} - 3.492 \times 10^{2} + 3.159 \times 10^{-1} =$

Answer Key:

1) $1.34 \times 10^{14} + 1.3 \times 10^{12} = (134 + 1.3) \times 10^{12} =$ **1.353×10^{14}**

2) $9.7821 \times 10^{-17} + 3.14 \times 10^{-18} = (97.821 + 3.14) \times 10^{-18} =$ **1.00961×10^{-16}**

3) $4.29754 \times 10^{3} + 1.34 \times 10^{2} = (42.9754 + 1.34) \times 10^{2} =$ **4.43154×10^{3}**

4) $7.523 \times 10^{25} - 6.32 \times 10^{22} + 1.34 \times 10^{24} = (7523 - 6.32 + 134) \times 10^{22} =$ **7.65068×10^{25}**

5) $6.5 \times 10^{-67} - 3.1 \times 10^{-65} = (6.5 - 310) \times 10^{-67} =$ **-3.035×10^{-65}**

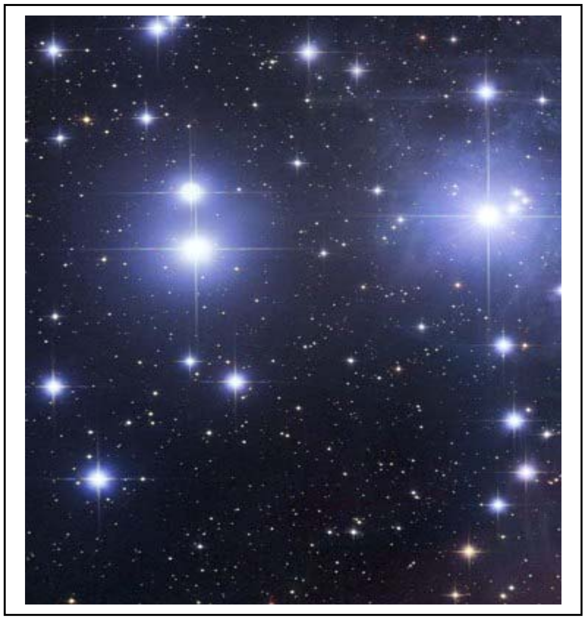
6) $3.872 \times 10^{11} - 2.874 \times 10^{13} = (3.872 - 287.4) \times 10^{11} =$ **2.83528×10^{13}**

7) $8.713 \times 10^{-15} + 8.713 \times 10^{-17} = (871.3 + 8.713) \times 10^{-17} =$ **8.80013×10^{-15}**

8) $1.245 \times 10^{2} - 5.1 \times 10^{-1} = (1245.0 - 5.1) \times 10^{-1} =$ **1.2399×10^{2}**

9) $3.64567 \times 10^{137} - 4.305 \times 10^{135} + 1.856 \times 10^{136} = (364.567 - 4.305 + 18.56) \times 10^{135} =$
3.78822×10^{137}

10) $1.765 \times 10^{4} - 3.492 \times 10^{2} + 3.159 \times 10^{-1} = (17650.0 - 3492 + 3.159) \times 10^{-1} =$
1.4161159×10^{4}



The following problems involve the multiplication and division of numbers expressed in Scientific Notation. Report all answers to two significant figures. For example:

$1.34 \times 10^8 \times 4.5 \times 10^6 = (1.34 \times 4.5) \times 10^{(8+6)}$
$= 6.03 \times 10^{14}$
To 2 significant figures this becomes… 6.0×10^{14}

$3.45 \times 10^{-5} / 2.1 \times 10^6 = (3.45/2.1) \times 10^{(-5 - (6))}$
$= 1.643 \times 10^{-11}$
To 2 significant figures this becomes… 1.6×10^{14}

1) Number of nuclear particles in the sun: 2.0×10^{33} grams / 1.7×10^{-24} grams/particle

2) Number of stars in the visible universe: 2.0×10^{11} stars/galaxy x 8.0×10^{10} galaxies

3) Age of universe in seconds: 1.4×10^{10} years x 3.156×10^7 seconds/year

4) Number of electron orbits in one year: (3.1×10^7 seconds/year) / (2.4×10^{-24} seconds/orbit)

5) Energy carried by visible light: (6.6×10^{-27} ergs/cycle) x 5×10^{14} cycles

6) Lengthening of Earth day in 1 billion years: (1.0×10^9 years) x 1.5×10^{-5} sec/year

7) Tons of TNT needed to make crater 100 km across: $4.0 \times 10^{13} \times (1.0 \times 10^{15})/(4.2 \times 10^{16})$

8) Average density of the Sun: 1.9×10^{33} grams / 1.4×10^{33} cm^3

9) Number of sun-like stars within 300 light years: (2.0×10^{-3} stars) x 4.0×10^6 cubic light-yrs

10) Density of the Orion Nebula: ($3.0 \times 10^2 \times 2.0 \times 10^{33}$ grams) / (5.4×10^{56} cm^3)

Answer Key:

1) Number of nuclear particles in the sun: 2.0×10^{33} grams / 1.7×10^{-24} grams/particle

1.2×10^{57} particles (protons and neutrons)

2) Number of stars in the visible universe: 2.0×10^{11} stars/galaxy x 8.0×10^{10} galaxies

1.6×10^{22} stars

3) Age of universe in seconds: 1.4×10^{10} years x 3.156×10^{7} seconds/year

4.4×10^{17} seconds

4) Number of electron orbits in one year: (3.1×10^{7} seconds/year) / (2.4×10^{-24} seconds/orbit)

1.3×10^{31} orbits of the electron around the nucleus

5) Energy carried by visible light: (6.6×10^{-27} ergs/cycle) x 5×10^{14} cycles

3.3×10^{-12} ergs

6) Lengthening of Earth day in 1 billion years: (1.0×10^{9} years) x 1.5×10^{-5} sec/year

1.5×10^{4} seconds or 4.2 hours longer

7) Tons of TNT needed to make crater 100 km across: $4.0 \times 10^{13} \times (1.0 \times 10^{15})/(4.2 \times 10^{16})$

9.5×10^{11} tons of TNT (equals 950,000 hydrogen bombs!)

8) Average density of the Sun: 1.9×10^{33} grams / 1.4×10^{33} cm^3

1.4 grams/cm^3

9) Number of sun-like stars within 300 light years: (2.0×10^{-3} stars) x 4.0×10^{6} cubic light-yrs

8.0×10^{3} stars like the sun.

10) Density of the Orion Nebula: ($3.0 \times 10^{2} \times 2.0 \times 10^{33}$ grams) / (5.4×10^{56} cm^3)

1.1×10^{-21} grams/cm^3

Evaluating and Graphing Polynomials

The Cat's Eye nebula (NGC 6543) imaged by the Hubble Space Telescope. At its center is a young white dwarf star located 11,000 light years from Earth.

In 7 billion years, our sun will become a red giant, shedding its atmosphere as a planetary nebula, and leaving behind its dense core. This core, about the size of Earth, is what astronomers call a white dwarf, and lacking the ability to create heat through nuclear reactions, it will steadily cool and become fainter as a stellar remnant.

The luminosity, L, of the white dwarf sun has been mathematically modeled as a function of time, t, to give **y = $\mathbf{Log_{10}L(t)}$** and **x = $\mathbf{Log_{10}t}$,** where t is in units of years and L is in multiples of the current solar power (3.8 x 10^{26} watts. The domain of the function is [+3.8, +10.5].

$$y(x) = 0.0026x^5 - 0.1075x^4 + 1.6895x^3 - 12.742x^2 + 45.396x - 59.024$$

Problem 1 - The domain over which y(x) applies as an approximation is given by the logarithmic interval [+3.8, +10.4]. Over what span of years does this correspond?

Problem 2 - Graph y(x) over the stated domain using a graphing calculator or Excel spreadsheet.

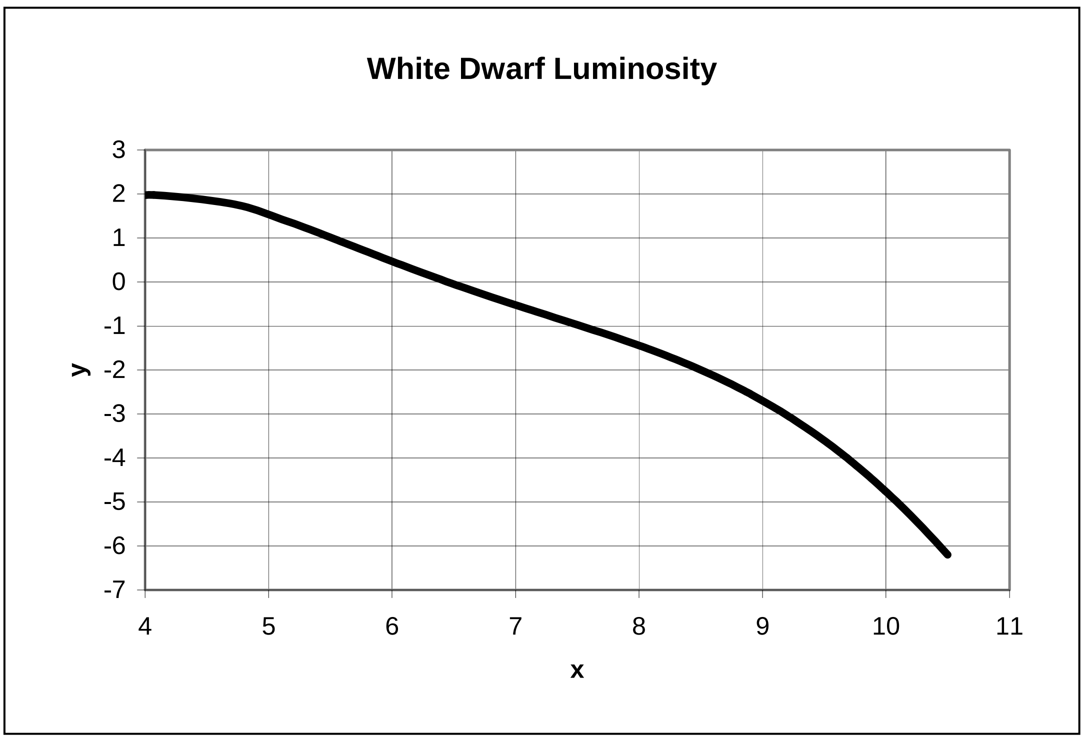
Problem 3 - For what values of t in years does y=0, and how is this physically interpreted in terms of L and t? [Hint: Use a calculator and make repeated guesses for x - called the Method of Successive Approximation]

Problem 1 - Answer: The text states that x = $\log_{10}t$ where t is in years, so +3.8 = $\log_{10}t$, and t = $10^{+3.8}$ years or 6,300 years. The upper bound is then +10.4 = $\log_{10}t$, and t = $10^{+10.4}$ years or 2.5 x 10^{10} years. **So the span is from 6,300 years to 25 billion years.**

Problem 2 - Answer: See graph below. Use all significant figures in stated polynomial coefficients!

Problem 3 - Answer: Students can bracket this 'zero' by trial and error **near x=6.4** (y=+0.05) or more accurately **between x=6.45** (y = +0.002) **and x = 6.46** (y = -0.007). For x=6.4, t = $10^{+6.4}$ = 2.5 million years and L = $10^{0.0}$ = 1 Lsun, so **after cooling for 6.4 million years, the white dwarf emits as much power, L, as the sun.**

Note to Teacher: This model is based on a detailed computer calculation by astronomers Iben and Tutukov in 1985, summarized in the research article 'Cooling of a White Dwarf' by D'Antona and Mazzitelli published in the Annual Reviews of Astronomy and Astrophysics, 1990, Volume 28, pages 139-181 Table 2, columns1 and 2.



Evaluating and Graphing Polynomials

The search is on for an important theoretical particle called the Higgs Boson at the Large Hadron Collider, which began operation on November 23, 2009. The mass of the Higgs Boson is actually not constant, but depends on the amount of energy that is used to create it. This remarkable behavior can be described by the properties of the following equation:

$$V(x) = 2x^4 - (1 - T^2)x^2 + \frac{1}{8}$$

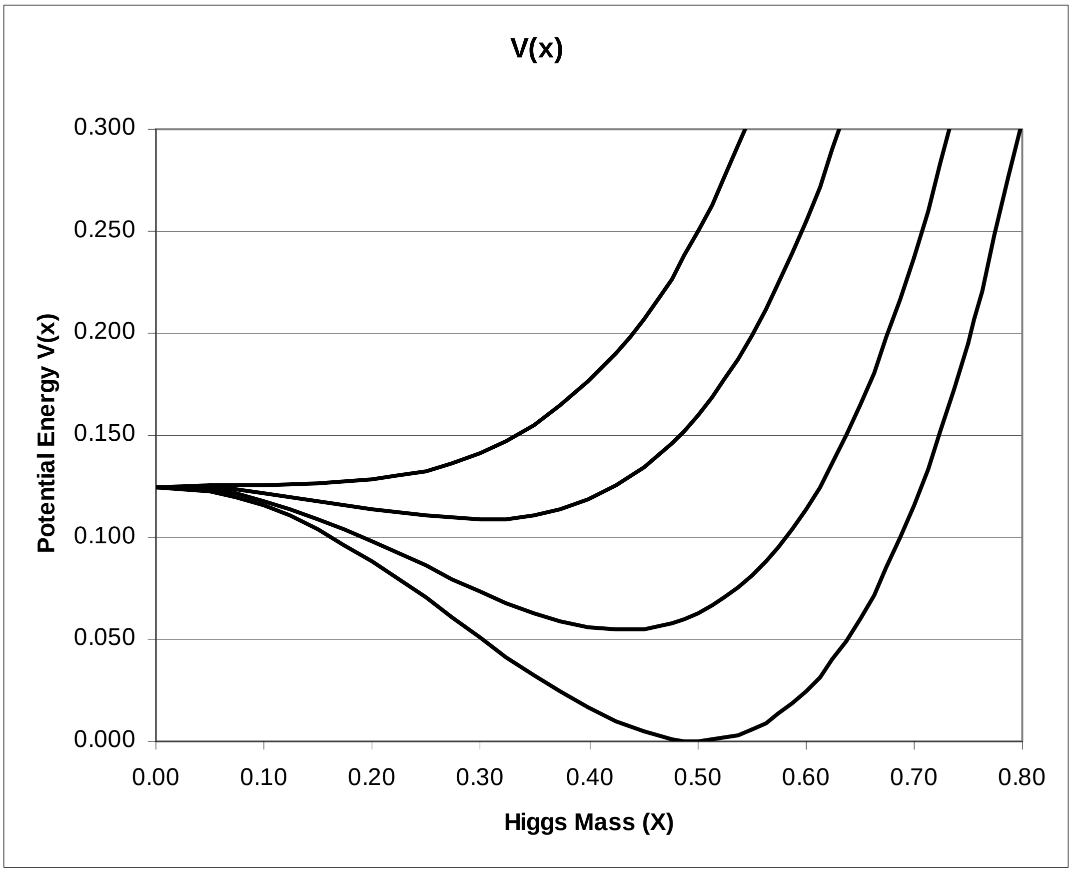
This equation describes the potential energy, V, stored in the field that creates the Higgs Boson. The variable x is the mass of the Higgs Boson, and T is the collision energy being used to create this particle. The Higgs field represents a new 'hyper-weak' force in Nature that is stronger than gravity, but weaker than the electromagnetic force. The Higgs Boson is the particle that transmits the Higgs field just as the photon is the particle that transmits the electromagnetic field.

Problem 1 - Using a graphing calculator, what is the shape of the function V(x) over the domain [0,+1] for a collision energy of; A) T=0? B) T=0.5? C) T = 0.8 and D) T=1.0?

Problem 2 - The mass of the Higgs Boson is defined by the location of the minimum of V(x) over the domain [0, +1]. If the mass, M in GeV, of the Higgs Boson is defined by **M = 300x**, how does the predicted mass of the Higgs Boson change as the value of T increases from 0 to 1?

Problem 1 - Answer: The function can be programmed on an Excel spreadsheet or a graphing calculator. Select x intervals of 0.05 and a graphing window of x: [0,1] y:[0,0.3] to obtain the plot to the left below. The curves from top to bottom are for T = 1, 0.8, 0.5 and 0 respectively.

Problem 2 - Answer: The minima of the curves can be found using a graphing calculator display or by interpolating from the spreadsheet calculations. The x values for T = 1 ,0.8, 0.5 and 0 are approximately 0, 0.3, 0.45 and 0.5 so the predicted Higgs Boson masses from the formula M = 300x will be 0 Gev, 90 GeV, 135 GeV and 150 GeV respectively.



Evaluating and Graphing Polynomials

An important concept in cosmology is that the 'empty space' between stars and galaxies is not really empty at all! Today, the amount of invisible energy hidden in space is just enough to be detected as Dark Energy, as astronomers measure the expansion speed of the universe. Soon after the Big Bang, this Dark Energy caused the universe to expand by huge amounts in less than a second. Cosmologists call this early period of the Big Bang Era, Cosmic Inflation.

An interesting property of this new 'dark energy' field, whose energy is represented by the function V(x), is that the shape of this function changes as the temperature of the universe changes. The result is that the way that this field, represented by the variable x, interacts with the other elementary particles in nature, changes. As this change from very high temperatures (T=1) to very low temperatures (T=0) occurs, the universe undergoes Cosmic Inflation!

$$V(x) = 2x^4 - (1 - T^2)x^2 + \frac{1}{8}$$

Problem 1 - What are the domain and range of the function V(x)?

Problem 2 - What is the axis of symmetry of V(x)?

Problem 3 - Is V(x) an even or an odd function?

Problem 4 - For T=0, what are the critical points of the function in the domain [-2, +2]?

Problem 5 - Over the domain [0,+2] where are the local minima and maxima located for T=0?

Problem 6 - Using a graphing calculator or an Excel spreadsheet, graph V(x) for the values T=0, 0.5, 0.8 and 1.0 over the domain [0,+1]. Tabulate the x-value of the local minimum as a function of T. In terms of its x location, what do you think happens to the end behavior of the minimum of V(x) in this domain as T increases?

Problem 7 - What is the vacuum energy difference V = V(0) - V(1/2) during the Cosmic Inflation Era?

Problem 8 - The actual energy stored in 'empty space' given by V(x) has the physical units of the density of energy in multiples of 10^{35} Joules per cubic meter. What is the available energy density during the Cosmic Inflation Era in these physical units?

Problem 1 - **Answer: Domain [- infinity, + infinity], Range [0,+infinity]**

Problem 2 - **Answer: The y-axis: x=0**

Problem 3 - **Answer: It is an even function.**

Problem 4 - **Answer: X = 0, X = -1/2 and x = +1/2**

Problem 5 - **Answer: The local maximum is at x=0; the local minimum is at x = +1/2**

Problem 6 - **Answer: See the graph below where the curves represent from top to bottom, T = 1.0, 0.8, 0.5 and 0.0. The tabulated minima are as follows:**

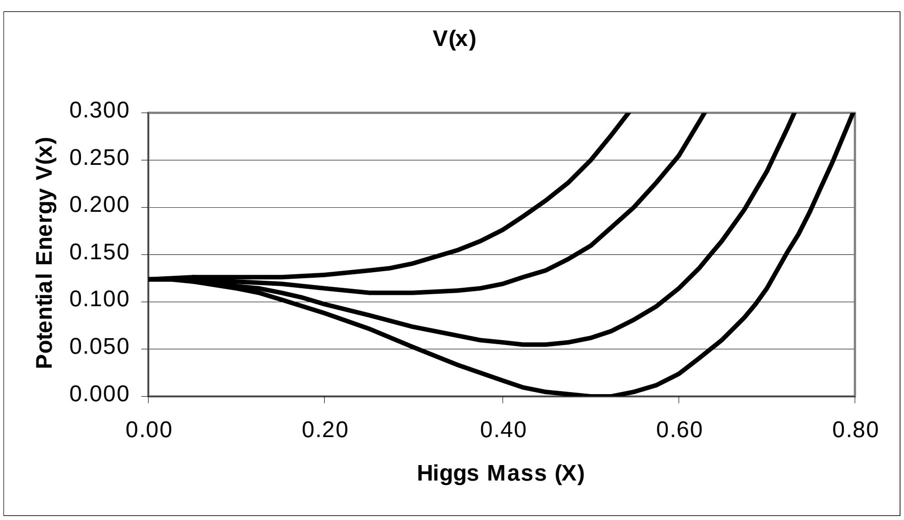
T	X
0.0	0.5
0.5	0.45
0.8	0.30
1.0	0.0

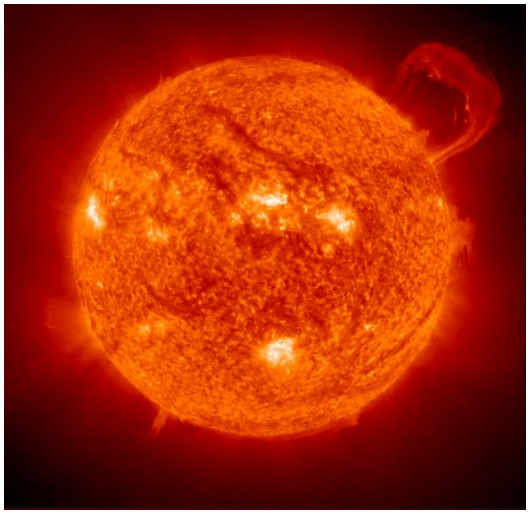
The end behavior, in the limit where T becomes very large, is that V(x) becomes a parabola with a vertex at (0, +1/8)

Problem 7 - What is the vacuum energy difference V = V(0) - V(1/2) during the Cosmic Inflation Era? **Answer: V(0) = 1/8 V(1/2) = 0 so V = 1/8.**

Problem 8 - The actual energy stored in 'empty space' given by V(x) has the physical units of the density of energy in multiples of 10^{35} Joules per cubic meter. What is the available energy density during the Cosmic Inflation Era in these physical units?
Answer: V = 1/8 x 10^{35} Joules/meter^3 = **1.2 x 10^{34} Joules/meter^3**.

Note to Teacher: This enormous energy was available in every cubic meter of space that existed soon after the Big Bang, and the time it took the universe to change from the V(0) to V(1/2) state lasted only about 10^{-35} seconds. This was enough time for the universe to grow by a factor of 10^{35} times in its size during the Cosmic Inflation Era.





Detailed mathematical models of the interior of the sun are based on astronomical observations and our knowledge of the physics of stars. These models allow us to explore many aspects of how the sun 'works' that are permanently hidden from view.

The Standard Model of the sun, created by astrophysicists during the last 50 years, allows us to investigate many separate properties. One of these is the density of the heated gas throughout the interior. The function below gives a best-fit formula, D(x) for the density (in grams/cm^3) from the core (x=0) to the surface (x=1) and points in-between.

$$D(x) = 519x^4 - 1630x^3 + 1844x^2 - 889x + 155$$

For example, at a radius 30% of the way to the surface, x = 0.3 and so D(x=0.3) = 14.5 grams/cm^3.

Problem 1 - What is the estimated core density of the sun?

Problem 2 - To the nearest 1% of the radius of the sun, at what radius does the density of the sun fall to 50% of its core density at x=0? (Hint: Use a graphing calculator and estimate x to 0.01)

Problem 3 - What is the estimated density of the sun near its surface at x=0.9 using this polynomial approximation?

Problem 1 - Answer; At the core, x=0, do D(0) = 155 grams/cm^3.

Problem 2 - Answer: We want D(x) = 155/2 = 77.5 gm/cm3. Use a graphing calculator, or an Excell spreadsheet, to plot D(x) and slide the cursor along the curve until D(x) = 77.5 .Then read out the value of x. The relevant portion of D(x) is shown in the table below:

X	D(x)
0.08	94.87
0.09	88.77
0.1	82.96
0.11	**77.43**
0.12	72.16
0.13	67.16
0.14	62.41

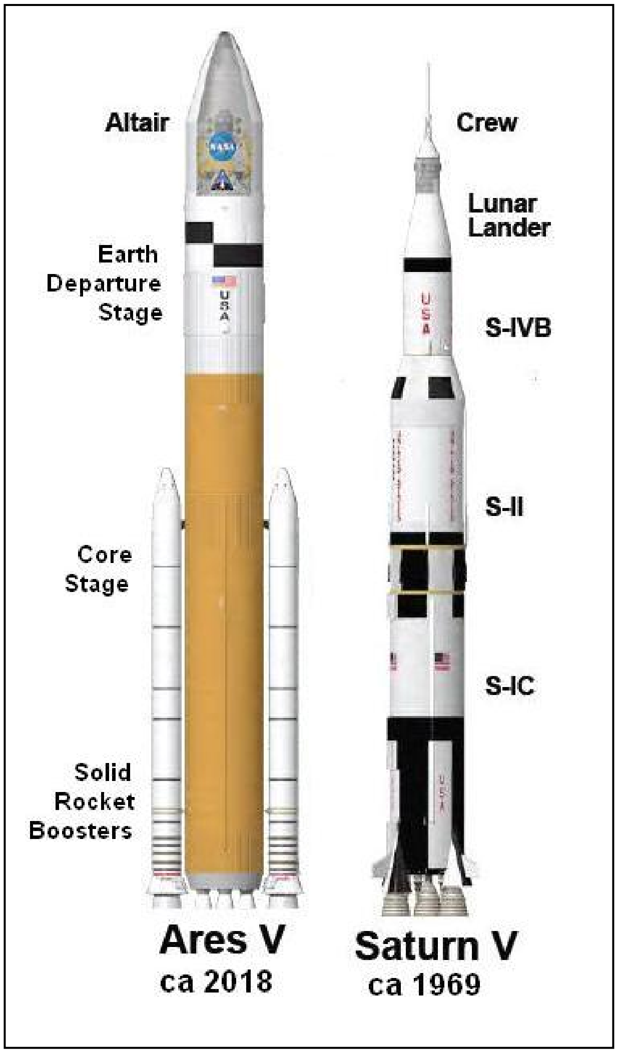
Problem 3 - Answer: At x=0.9 (i.e. a distance of 90% of the radius of the sun from the center).

$D(0.9) = 519(0.9)^4 - 1630(0.9)^3 + 1844(0.9)^2 -889(0.9) + 155$

$D(0.9) = 340.516 - 1188.27 + 1493.64 - 800.10 + 155.00$

D(0.9) = 0.786 gm/cm^3.

Multiplying and Dividing Polynomials



The Ares-V rocket, now being developed by NASA, will weigh 3,700 tons at lift-off, and be able to ferry 75 tons of supplies, equipment and up to 4 astronauts to the moon. As a multi-purpose launch vehicle, it will also be able to launch complex, and very heavy, scientific payloads to Mars and beyond. To do this, the rockets on the Core Stage and Solid Rocket Boosters (SRBs) deliver a combined thrust of 47 million Newtons (11 million pounds). For the rocket, let's define:

T(t) = thrust at time-t
m(t) = mass at time-t
a(t) = acceleration at time-t

so that:

$$a(t) = \frac{T(t)}{m(t)}$$

The launch takes 200 seconds. Suppose that over the time interval [0,200], T(t) and m(t) are approximately given as follows:

$$T(x) = 8x^3 - 16x^2 - x^4 + 47$$

$$m(x) = 35 - x^2$$

where t = 40x

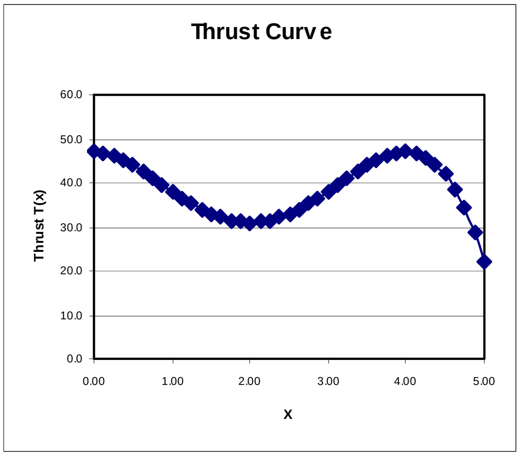
Where we have used a change of variable, from t to x to simplify the form of the equations.

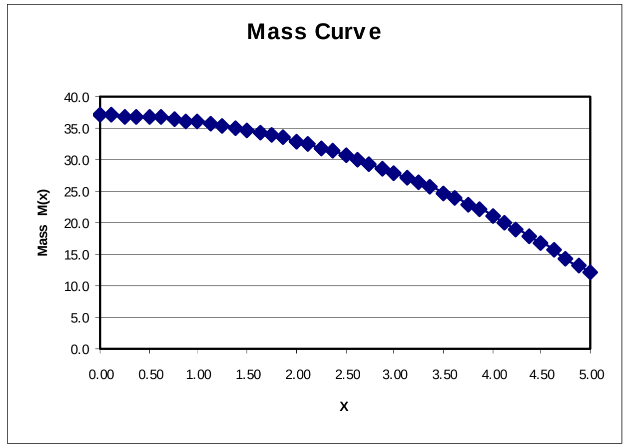
Problem 1 - Graph the thrust curve T(x), and the mass curve m(x) and find all minima, maxima inflection points in the interval [0,5]. (You may use a graphing calculator, or Excel spreadsheet.)

Problem 2 - Graph the acceleration curve a(x) and find all maxima, minima, inflection points in the interval [0,5]. (You may use a graphing calculator, or Excel spreadsheet.)

Problem 3 - For what value of x will the acceleration of the rocket be at its absolute maximum in the interval [0,5]? How many seconds will this be after launch? (Hint: You may use a graphing calculator, or Excel spreadsheet)

Answer Key



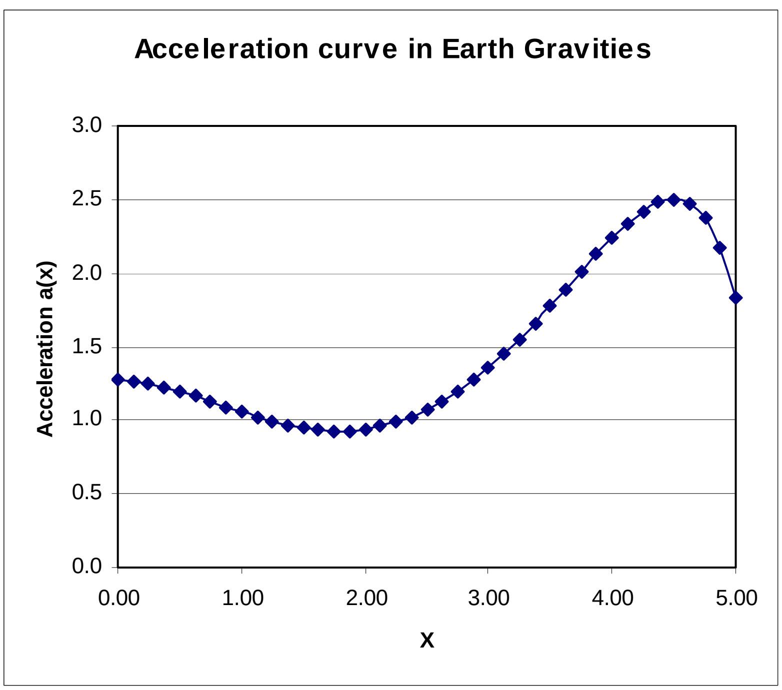


Problem 1 - The above graphs show T(x) and m(x) graphed with Excel. Similar graphs will be rendered using a graphing calculator. For the thrust curve, T(x), the relative maxima are at (0, 47) and (4,47). The relative minimum is at (2,31).

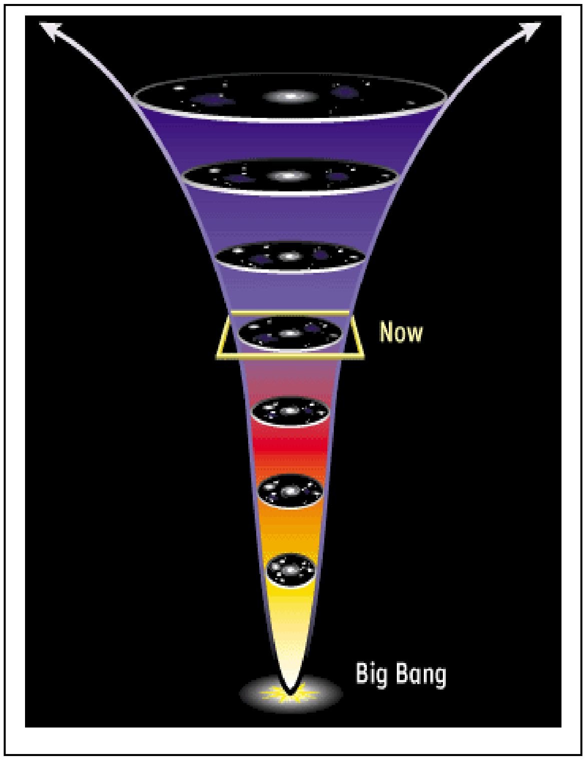
Problem 2 - For the mass curve, M(x), the absolute maximum is at (0,37).

Problem 3 - Answer: The curve reaches its maximum acceleration near (4.5,2.5). Because t = 40 X, this occurs about 40 x 4.5 = 180 seconds after launch.

Note to teacher: *The units for acceleration are in Earth Gravities (1 G = 9.8 meters/sec^2) so astronauts will feel approximately 2.5 times their normal weight at this point in the curve.*



Analyzing Graphs of Polynomial Functions



Since the 1930's, physicists have known that the 'vacuum' of space is not empty. It contains particles and energy that come and go, and cannot be directly detected. Moments after the Big Bang, this vacuum energy was large enough that, by itself, it was able to cause the universe to expand by trillions of times in size. Astronomers call this Cosmological Inflation.

A number of theoretical studies of the vacuum state have focused attention on a polynomial function:

$$V(x) = \frac{L}{6}x^4 - m^2x^2$$

This function, called the Coleman-Weinberg Potential, allows physicists to calculate the energy of the vacuum state, V(x), in terms of the mass, x, of a new kind of yet-to-be-discovered particle called the X-Boson.

Problem 1 – Factor V(x) and determine the location for all of the x-intercepts for the general case where m and L are not specified.

Problem 2 – For the specific case of V(x) for which m=5 and L = 6, determine its x-intercepts.

Problem 3 – Graph V(x) for m=5 and L=6 by plotting a selection of points between the x-intercepts.

Problem 4 – What is the end behavior of V(x) for the selected values of m and L?

Problem 5 – Use a graphing calculator to find the relative maximum and the relative minima for V(x) with m=5 and L=6.

Problem 1 – Factor V(x) and determine the location for all of the x-intercepts for the general case where m and L are not specified.

Answer: $V(x) = L/6\ x^2\ (x^2 – 6m^2/L)$

The x-intercepts, where V(x)=0 are x1=0,

$x2=+(6m^2/L)^{1/2}$ and $x3 = -(6m^2/L)^{1/2}$

Problem 2 – For the specific case of V(x) for which m=5 and L = 6, determine its x-intercepts.

Answer: The function is $V(x) = x^4 – 25x^2$ so

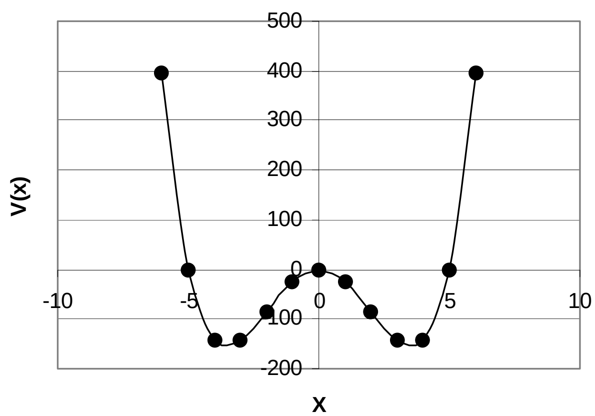
x1 = 0, $x2 = (25)^{1/2}$ = +5 and x3 = -5

Problem 3 – Graph V(x) for m=5 and L=6 by plotting a selection of points between the x-intercepts.

Answer: Below are some representative points:

x	-6	-4	-3	-2	-1	0	+1	+2	+3	+4	+6
V(x)	+396	-144	-144	-84	-24	0	-24	-84	-144	-144	+396

Sample graph:

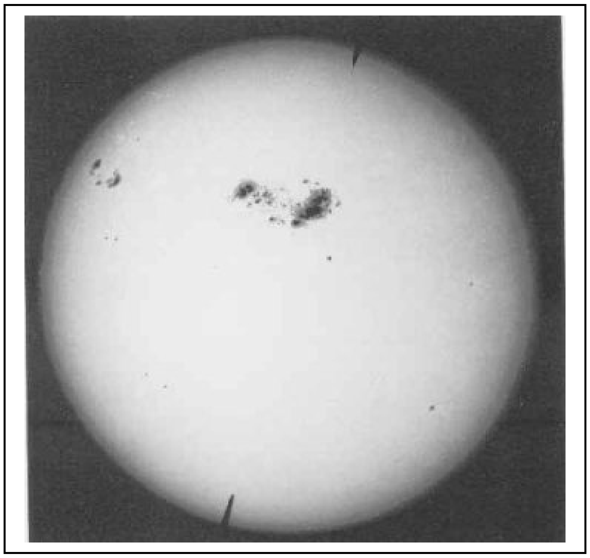


Problem 4 – What is the end behavior of V(x) for the selected values of m and L? Answer: For x < -5 V(x) remains positive and increases to +infinity. For x > +5 V(x) also remains positive and increases to + infinity.

Problem 5 – Use a graphing calculator to find the relative maximum and the relative minima for V(x) with m=5 and L=6. Answer: The relative maximum is at x=0, V(x)=0 The relative minima are near x= +3.5, V(x)= -156, and x = -3.5, V(x) = -156.

Note: The exact values for the relative minima, using calculus, are

$x = +/- (3M^2/L) = +/- 5(2)^{1/2} / 2 = +/- 3.54$ $V(x) = -3/2\ (M^4/L) = -156.25$.



Unlike planets or other solid bodies, the sun does not rotate at the same speed at the poles or equator. By tracking sunspots at different latitudes, astronomers can map out the 'differential rotation' of this vast, gaseous sphere.

The photo to the left shows a large sunspot group as it passes across the face of the sun to reveal over the course of a few weeks, the rotation rate of the sun.

(Carnegie Institute of Washington image)

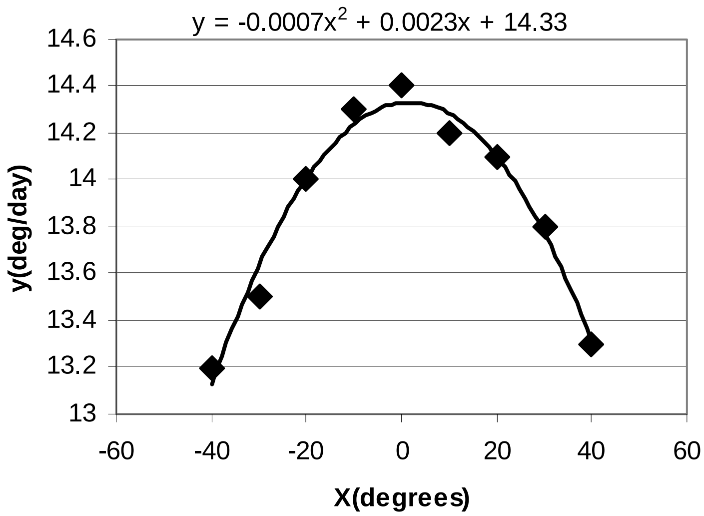
Problem 1 – The table below gives the speed of rotation, V, of the sun at different latitudes, X. Find a polynomial, V(x), that fits this data.

X	-40	-30	-20	-10	0	+10	+20	+30	+40
V	13.2	13.5	14.0	14.3	14.4	14.2	14.1	13.8	13.3

Problem 2 – To three significant figures, what would you predict as the speed of rotation at a latitude of -30?

Problem 3 – The physical units for V(x) are degrees per day so that, for example, V(-20) = 14.0 degrees per day. From your answer to Problem 1, create a related function, P(x), that predicts the rotation period of the Sun in terms of the number of days it takes to make a complete 360-degree rotation at each latitude. To three significant figures, A) how many days does it take at the equator (x=0)? B) how many days does it take at a latitude of +40 degrees?

Problem 1 Answer: Using an Excel spreadsheet, a best-fit quadratic function is $V(x)=-0.0007x^2 + 0.0023x +14.33$



Problem 2 – To three significant figures, what would you predict as the speed of rotation at a latitude of -30?

Answer: $V(50)= -0.0007(-30)^2 + 0.0023(-30) +14.33 = 13.631$

which to three significant figures is just **13.6 degrees/day**.

Problem 3 – The physical units for V(x) are degrees per day so that, for example, V(-20) = 14.0 degrees per day. From your answer to Problem 1, create a related function, P(x), that predicts the rotation period of the Sun in terms of the number of days it takes to make a complete 360-degree rotation at each latitude. To three significant figures, A) how many days does it take at the equator (x=0)? B) how many days does it take at a latitude of +40 degrees?

Answer: P(x) = 360 / V(x) so

$$P(x) = \frac{360}{(-0.0007x^2 + 0.0023x + 14.33)}$$

A) P(0) = 360/14.33 = **25.0 days**.

B) P(+40) = **27.1 days**

As a comet orbits the sun, it produces a long tail stretching millions of kilometers through space. The tail is produced by heated gases leaving the nucleus of the comet.

This image of the head of Comet Tempel-1 was taken by the Hubble Space Telescope on June 30, 2005. It shows the 'coma' formed by these escaping gases about 5 days before its closest approach to the sun (perihelion). The most interesting of these ingredients is ordinary water.

Problem 1 – The table below gives the number of tons of water produced every minute, W, as Comet Tempel-1 orbited the sun. Find a polynomial, W(T), that fits this data, where T is the number of days since its closest approach to the sun, called perihelion.

T	-120	-100	-80	-60	-40	-20	0	+20	+40
W	54	90	108	135	161	144	126	54	27

Problem 2 – To two significant figures, how many tons of water each minute were ejected by the comet 130 days before perihelion (T = -130)?

Problem 3 - To two significant figures, determine how many tons of water each minute were ejected by the comet 50 days after perihelion (T = +50). Can you explain why this may be a reasonable prediction consistent with the mathematical fit, yet an implausible 'Real World' answer?

Problem 1 – The table below gives the number of tons of water produced every minute, W, as Comet Tempel-1 orbited the sun. Find a polynomial, W(T), that fits this data, where T is the number of days since its closest approach to the sun, called perihelion.

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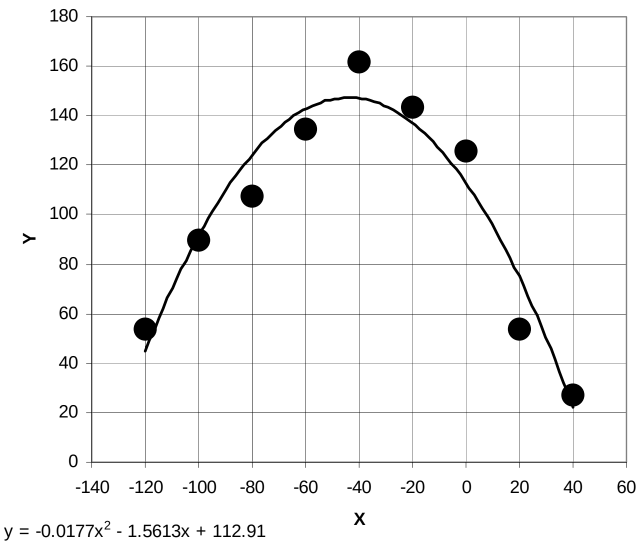
W	54	90	108	135	161	144	126	54	27

Answer: The graph below was created with Excel, and a quadratic trend line was selected. The best fit was for W(T)=-0.0177x2 – 1.5613x +112.91. Graphing calculators may produce different fits depending on the polynomial degree used.

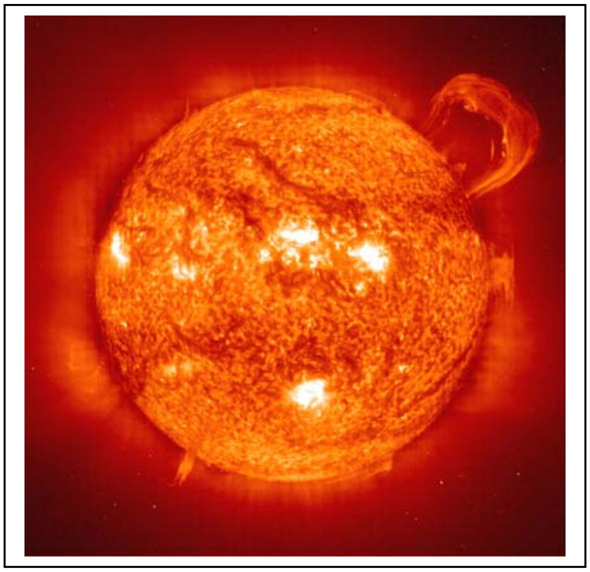


Problem 2 – To two significant figures, how many tons of water each minute were ejected by the comet 130 days before perihelion (T = -130)?

Answer: From the fitted polynomial above
$W(-130) = -0.0177(-130)^2 - 1.5613(-130) + 112.91$ = **17 tons/minute**

Problem 3 - To two significant figures, determine how many tons of water each minute were ejected by the comet 50 days after perihelion (T = +50). Can you explain why this may be a reasonable prediction consistent with the mathematical fit, yet an implausible 'Real World' answer?

Answer: The fitting function W(T) predicts that W(50) = -9.4 tons per minute. Although this value smoothly follows the prediction curve, it implies that instead of ejecting water (positive answer means a positive rate of change) the comet is absorbing water (negative answer means a negative rate of change), so the prediction is not realistic.



The sun is an active star. Matter erupts from its surface and flows into space under the tremendous magnetic forces at play on its surface.

Among the most dramatic phenomena are the eruptive prominences, which eject billions of tons of matter into space, and travel at thousands of kilometers per minute.

This image from the Solar and Heliophysics Observatory (SOHO) satellite taken on September 23, 1999 and shows a giant prominence being launched from the sun.

Problem 1 – The table below shows the height versus time data for an eruptive prominence seen on August 6, 1931. Graph the data, and find a polynomial, h(t), that fits this data.

t	**15**	**15.5**	**16**	**16.5**	**17**	**17.5**	**18**	**18.5**	**19**
h	**50**	**60**	**70**	**100**	**130**	**150**	**350**	**550**	**700**

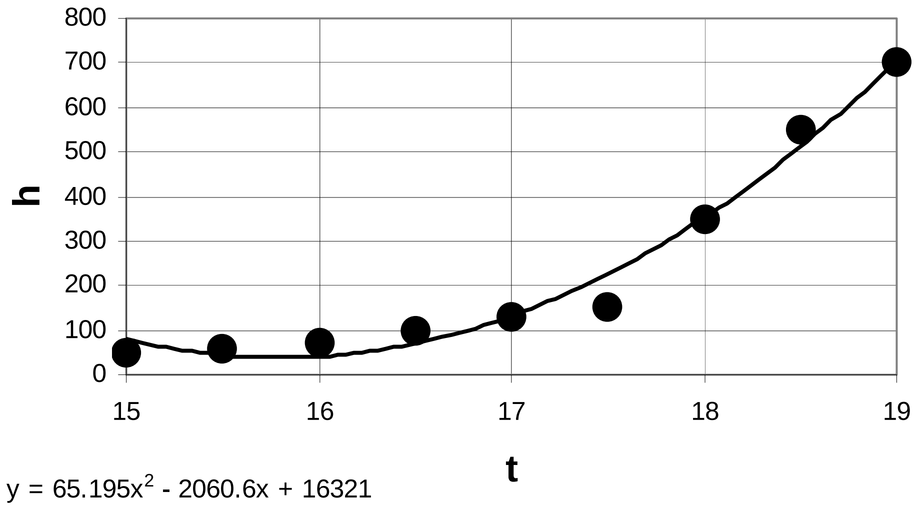
Problem 2 – The data give the height, h, of the eruptive prominence in multiples of 1,000 kilometers from the solar surface, for various times, t, given in hours. For example, at a time of 17 hours, the prominence was 130,000 kilometers above the solar surface. To two significant figures, how high was the prominence at a time of 19.5 hours?

Problem 1 – The table below shows the height versus time data for an eruptive prominence seen on August 6, 1931. Graph the data, and find a polynomial, h(t), that fits this data.

t	15	15.5	16	16.5	17	17.5	18	18.5	19
h	**50**	**60**	**70**	**100**	**130**	**150**	**350**	**550**	**700**

Answer: The best fit degree-2 polynomial is

$$h(t)= 65.195t^2 – 2060\ t +16321$$



Problem 2 – The data give the height, h, of the eruptive prominence in multiples of 1,000 kilometers from the solar surface, for various times, t, given in hours. For example, at a time of 17 hours, the prominence was 130,000 kilometers above the solar surface. To two significant figures, how high was the prominence at a time of 19.5 hours?

Answer: $h = 16.195(19.5)^2 – 2060.6\ (19.5) + 16321$
$= 941.398$
$= 940$ to two significant figures

Since h is in multiples of 1,000 km, the answer will be **940,000 kilometers**.