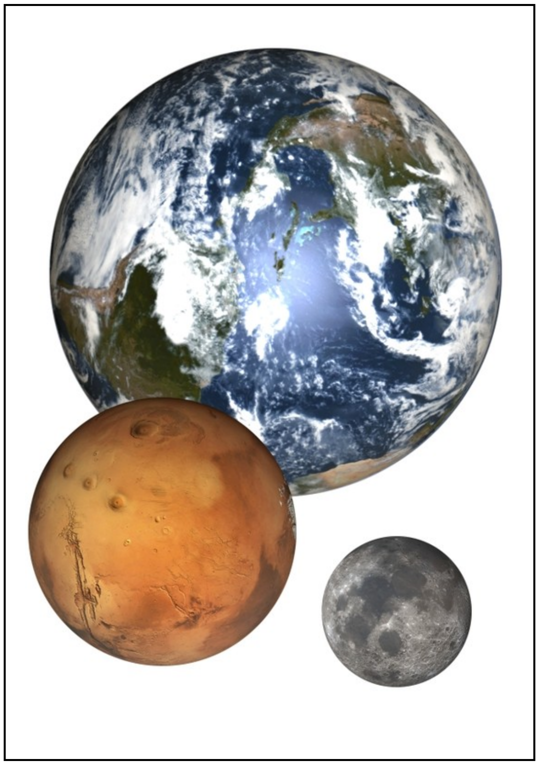


'What goes up, must come down', is a common expression that can be represented by a quadratic equation! If you were to plot the height of a ball tossed vertically, its height in time would follow a simple quadratic formula in time given by the general equation:

$$H(t) = h_0 + Vt - \frac{1}{2}gt^2$$

where h_0 is the initial height of the ball in meters, V is the initial speed in meters/second, and g is the acceleration of gravity in meters/second^2. It is a general equation because it works, not only on Earth, but also on nearly all other astronomical bodies, except for black holes! For black holes, the geometry of space is so distorted that t, V and h_0 are altered in complex ways.

For the following problems: A) write the equation in Standard Form, B) determine the coordinates of the vertex of the parabola where H(t) is a maximum; C) determine the axis of symmetry; D) On a common graph for all three problems, draw the parabola for each problem by plotting two additional points using the property of the axis of symmetry, for all positive times during which H(t) > 0

Problem 1 – On Earth, the acceleration of gravity is g = 10 meters/sec^2. The ball was thrown vertically at an initial speed of V=20 meters/sec (45 mph) from a height of h_0 = 2 meters.

Problem 2 – On Mars, the acceleration of gravity is g = 4 meters/sec^2. The ball was thrown vertically at an initial speed of V=20 meters/sec (45 mph) from a height of h_0 = 2 meters.

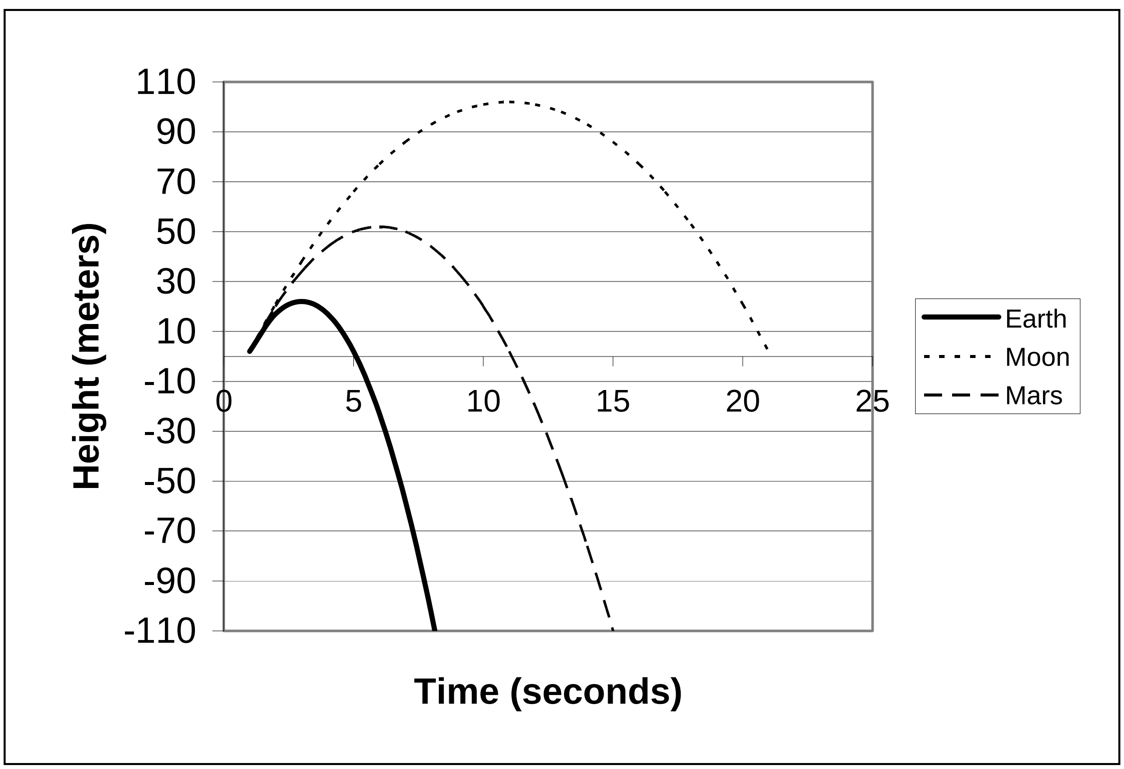
Problem 3 – On the Moon, the acceleration of gravity is g = 2 meters/sec^2. The ball was thrown vertically at an initial speed of V=20 meters/sec (45 mph) from a height of h_0 = 2 meters.

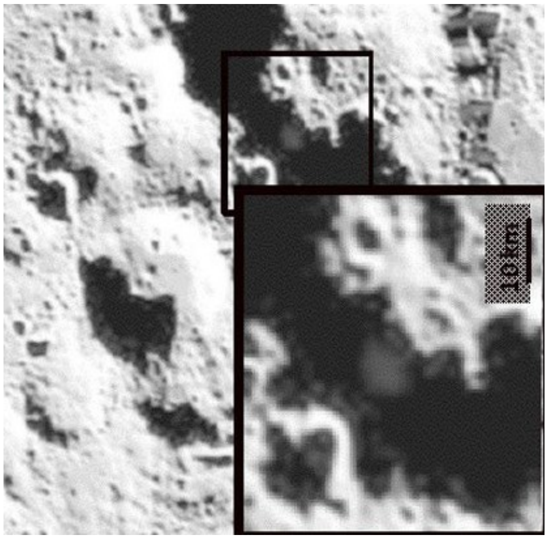
Problem 1 –Answer: Standard Form is $H(t) = at^2 + bt + c$
A) $H(t) = -5t^2 + 20t + 2$
B) $(t,H) = (-b/2a, c - b^2/4a)$ so t= 2 seconds and H(2) = 22 meters. Vertex: **(+2,+22)**
C) The line t = 2
D) See graph below

Problem 2 – Answer: Standard Form is $H(t) = at^2 + bt + c$
A) $H(t) = -2t^2 + 20t + 2$
B) $(t,H) = (-b/2a, c - b^2/4a)$ so t=5 seconds and H(5) = 52 meters. Vertex **(+5,+52)**
C) The line t = 5
D) See graph below

Problem 3 – Answer: Standard Form is $H(t) = at^2 + bt + c$
A) $H(t) = -t^2 + 20t + 2$
B) $(t,H) = (-b/2a, c- b^2/4a)$ so t=10 seconds and H(2) = 102 meters. Vertex **(+10,+102)**
C) The line t = 10
D) See graph below

Note: This would be a good opportunity to emphasize that 1) the plotted graph is not the trajectory of the ball in 2-dimensions…a common misconception. 2) negative values of H(t) are for elevations below the zero point of the launch altitude are unphysical, as are the portions of the curves for T < 0 'before' the ball was thrown. Sometimes math models of physical phenomena can lead to unphysical solutions over part of the domain/range of interest and have to be interpreted against the physical world to understand what they mean.





LCROSS debris plume seen in the dark Shadow of a lunar crater. (NASA/LRO)

After a meteorite strikes the surface of a planet, the debris fragments follow parabolic trajectories as they fall back to the surface. When the LCROSS impactor struck the moon, debris formed a plume of material that reached a height of 10 kilometers, and returned to the surface surrounding the crater.

The size of the debris field surrounding the crater can be estimated by solving a quadratic equation to determine the properties of the average trajectory of the debris. The equation that approximates the average particle trajectory is given by

$$H(x) = x - \frac{g}{2V^2}x^2$$

Problem 1 – The equation gives the height, H(x) in meters, of an average particle ejected a distance of x from the impact site for which V is the speed of the particles in meters/sec and g is the acceleration of gravity on the surface of the Moon in meters/sec^2. Factor this equation to find its two 'roots', which represent the initial ejection distance from the center of the impact crater, and the final landing distance of the particles from the center of the crater.

Problem 2 – At what distance from the center of the crater did the debris reach their maximum altitude?

Problem 3 – What was the maximum altitude of the debris along their trajectory?

Problem 4 – Solve this parabolic equation for the specific case of the LCROSS ejecta for which V = 200 meters/sec and g = 2 meters/sec^2 to determine A) the maximum radius of the debris field around the crater, and B) the maximum height of the debris plume.

Problem 1 – The equation gives the height, H(t) in meters, of an average particle ejected from the impact site for which V is the speed of the particles in meters/sec and g is the acceleration of gravity on the surface of the Moon in meters/sec^2. Factor this equation to find its two 'roots', which represent the initial ejection distance from the center of the impact crater, and the final landing distance of the particles from the center of the crater.

Answer: $H(t) = x - \frac{g}{2V^2}x^2$ can be factored to get $H(t) = (x-0)\left(1 - \frac{g}{2V^2}x\right)$

The first factor yields X=0 which represents the starting distance of the debris from the center of the crater. The second factor has a value of zero for x = (2V^2)/g

Problem 2 – At what distance from the center of the crater did the debris reach their maximum altitude?

Answer: By symmetry, the maximum height was reached half-way between the endpoints of the parabola or $x = \frac{1}{2}\left(\frac{2V^2}{g} - 0\right) = \frac{V^2}{g}$

Problem 3 – What was the maximum altitude of the debris along their trajectory?

Answer: Evaluate H(V^2/g) to get

$$H\left(\frac{V^2}{g}\right) = \left(\frac{V^2}{g}\right) - \frac{g}{2V^2}\left(\frac{V^2}{g}\right)^2 \quad \text{so} \quad H\left(\frac{V^2}{g}\right) = \frac{V^2}{2g}$$

Problem 4 – Solve this parabolic equation for the specific case of the LCROSS ejecta for which V = 200 meters/sec and g = 2 meters/sec^2 to determine A) the maximum radius of the debris field around the crater, and B) the maximum height of the debris plume.

Answer:
A) Maximum radius of ejecta field
x = 2V^2/g = 2 (200)^2/2 = **40,000 meters or 40 kilometers**.

B) Maximum height of debris plume
H=V^2/2g = (200)^2/4 = 40000/4 = **10,000 meters or 10 kilometers**.


Spectacular photo of a fighter jet breaking the 'sound barrier' at a speed of 741 mph or 331 meters/sec (Courtesy Ensign John Gay).

The speed of average air molecules (mostly nitrogen and oxygen) is related to the atmospheric temperature by the formula:

$$V^2 = 400\ T$$

where V is the speed in meters/sec and T is the gas temperature on the Kelvin scale.

This speed is important to know because it defines the speed of sound.

You can convert a temperature in degrees Fahrenheit (F) or Celsius (C) to Kelvins (T) by using the formulae:

$$T = \left(\frac{5}{9}\right)(F - 32) + 273 \quad \text{or} \quad T = C + 273$$

Problem 1 - On the coldest day in Vostok, Antarctica on July 21, 1983, the temperature was recorded as F = -128^{o} Fahrenheit; A) What was the temperature ,T, on the Kelvin scale?; B) What was the speed of the air molecules at this temperature in meters/second?

Problem 2 - At normal sea-level on an average day in Spring, the temperature is about F = +60^{o} Fahrenheit. What is the speed of sound at sea-level in meters/sec?

Problem 3 - The hottest recorded temperature on Earth was measured at El Azizia, Libya and reached +80^{o} Celsius; A) What was this temperature, T, in Kelvins ? B) What was the speed of sound at this location in meters/sec?

Problem 1 - On the coldest day in Vostok, Antarctica on July 21, 1983, the temperature was recorded as F = -128^o Fahrenheit; A) What was the temperature ,T, on the Kelvin scale?; B) What was the speed of the air molecules at this temperature in meters/second?

Answer: A) T = 5/9 (-128 - 32) + 273 so **T = 184 Kelvin**.

B) $V^2 = 400\ T$ so
$V^2 = 400\ (184)$
$V^2 = 73{,}600$

And taking the square-root of both sides we get
V = 271 meters/sec.

Problem 2 - At normal sea-level on an average day in Spring, the temperature is about F = $+60^o$ Fahrenheit. What is the speed of sound at sea-level in meters/sec?

Answer: A) T = 5/9 (60 - 32) + 273 so **T = 288 Kelvin**.

B) $V^2 = 400\ T$ so
$V^2 = 400\ (288)$
$V^2 = 115{,}200$

And taking the square-root of both sides we get
V = 339 meters/sec.

Problem 3 - The hottest recorded temperature on Earth was measured at El Azizia, Libya and reached $+80^o$ Centigrade; A) What was this temperature, T, in Kelvin degrees? B) What was the speed of sound at this location in meters/sec?

Answer: A) T = (80) + 273 so **T = 353 Kelvin**.

B) $V^2 = 400\ T$ so
$V^2 = 400\ (353)$
$V^2 = 141{,}200$

And taking the square-root of both sides we get
V = 376 meters/sec.

This dark cloud, called Thackeray's Globule is slowly collapsing, and in several million years may form one or more stars. It is currently about 2 light years across. (Courtesy: Hubble Space Telescope/NASA)

Clouds of gas in interstellar space cannot remain as they are for very long because the gravity from their own mass causes them to collapse under their own weight. The formula that relates the collapse time, T, to the density of the gas cloud, N, is:

$$T = \frac{2x10^{6}}{\sqrt{N}}$$

where **T** is the time in years and **N** is the density of the gas cloud given in atoms per cubic centimeter.

Problem 1 - A typical Bok Globule 'dark cloud' one light year in diameter and containing about 100 times of the mass of our sun in interstellar gas, and has a density of about 4000 atoms/cc. To one significant figure, what is its estimated collapse time in years?

Problem 2 - The Milky Way originally formed from a cloud of gas with an average density that may have been about 2 atoms/cc. To one significant figure, about how long did it take the Milky Way cloud to collapse?

Problem 1 - A typical Bok Globule 'dark cloud' 1 light year in diameter and containing about 100 times of the mass of our sun in interstellar gas, and has a density of about 4000 atoms/cc. To one significant figure, what is its estimated collapse time in years?

Answer; N = 4000 so from the formula:

$$T = \frac{2x10^6}{\sqrt{4000}}$$

T = 31,623 years, and to one significant figure this is **30,000 years**.

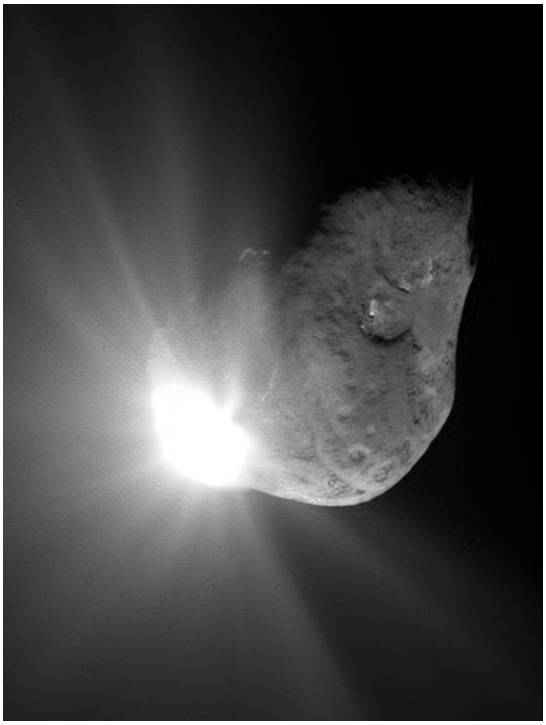
Problem 2 - The Milky Way originally formed from a cloud of gas with an average density that may have been about 2 atoms/cc. To one significant figure, about how long did it take the Milky Way cloud to collapse?

Answer:

Answer; N = 2 so from the formula:

$$T = \frac{2x10^6}{\sqrt{2}}$$

T = 1,414,000 years, and to one significant figure this is **1 million years**.



Clouds of debris are produced when a high-speed projectile slams into a moon or an asteroid. The image to the left, was taken by the Deep Impact spacecraft seconds after its 370-kg impactor struck the nucleus of Comet Tempel-1 in 2005.

Once at the peak of their trajectory, some of the debris particles fall back to the surface. Their height above the surface, H(t), is given by the function:

$$H(t) = h_0 - \frac{1}{2}gt^2$$

where h_0 is their starting altitude above the surface in meters, and g is the acceleration of gravity in meters/sec^2.

Problem 1 – On October 9, 2009, the LCROSS Impactor slammed into the surface of the Moon with the energy equal to about 2 tons of TNT, and produced a crater 350 meters across. The ejecta plume containing a mixture of heated lunar rock and trapped water, was observed to reach a height of 10 kilometers. If the acceleration of gravity on the surface of the Moon is 1.6 meters/sec^2, how long did it take for the highest plume particles to fall back to the lunar surface?

Problem 2 – On July 4, 2005, the Deep Impact spacecraft flew past Comet Tempel-1 and launched a 370 kg impactor that struck the surface of the comet at a speed of 10 kilometers/sec with an equivalent energy of about 5 tons of TNT. A bright plume of ejecta was observed, which contained a mixture of dust and water-ice particles. Although most of the debris particles were completely ejected during the impact, some slower-moving particles remained behind and eventually fell back to the comet's surface. If the highest altitude of the returning debris particles was about 300 meters, how long did it take for them to reach the surface if the acceleration of gravity for this small body was only 0.34 meters/sec^2?

Problem 1 – On October 9, 2009, the LCROSS Impactor slammed into the surface of the Moon with the energy equal to about 2 tons of TNT, and produced a crater 350 meters across. The ejecta plume containing a mixture of heated lunar rock and trapped water, was observed to reach a height of 10 kilometers. If the acceleration of gravity on the surface of the Moon is 1.6 meters/sec^2, how long did it take for the highest plume particles to fall back to the lunar surface?

Answer: In the equation, H(t) = 0 on the lunar surface, h_0 = 10,000 meters, and g = 1.6 meters/sec^2. So:

$$0 = 10{,}000 - 0.5\,(1.6)\,t^2$$
$$t^2 = 12{,}500$$

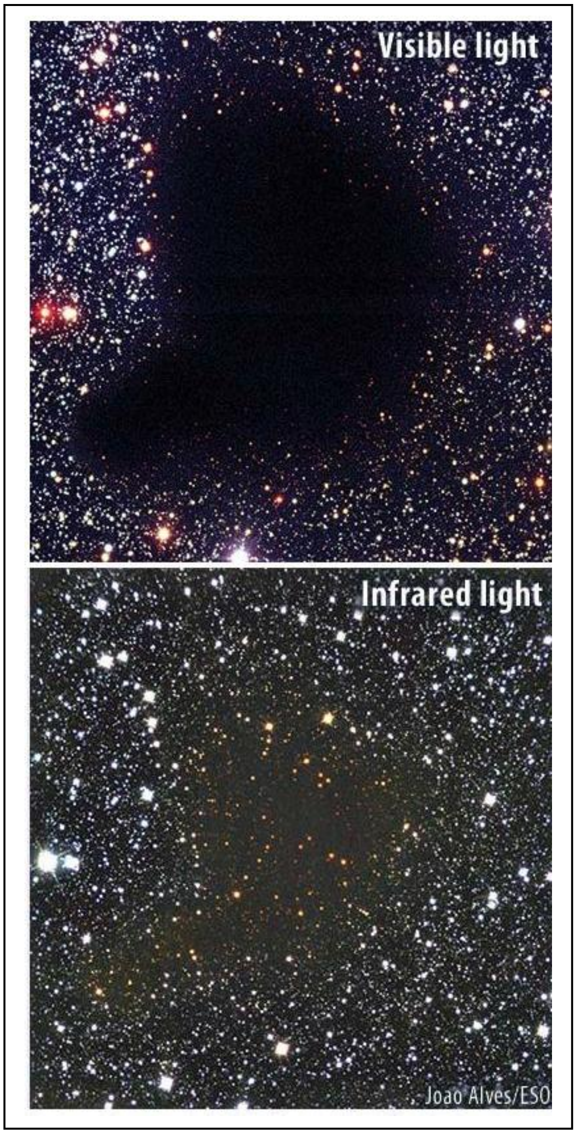
Taking the square-root we get t = +112 seconds or t = -112 seconds.
Since we are considering a time in the future of the present moment when h = 10,000 we eliminate the negative solution and get **t = +112 seconds**. This is nearly 2 minutes.

Problem 2 – On July 4, 2005, the Deep Impact spacecraft flew past Comet Tempel-1 and launched a 370 kg impactor that struck the surface of the comet at a speed of 10 kilometers/sec with an equivalent energy of about 5 tons of TNT. A bright plume of ejecta was observed, which contained a mixture of dust and water-ice particles. Although most of the debris particles were completely ejected during the impact, some slower-moving particles remained behind and eventually fell back to the comet's surface. If the highest altitude of the returning debris particles was about 300 meters, how long did it take for them to reach the surface if the acceleration of gravity for this small body was only 0.34 meters/sec^2?

Answer: In the equation, H(t) = 0 on the comet's surface, h_0 = 300 meters, and g = 0.34 meters/sec^2. So:

$$0 = 300 - 0.5\,(0.34)\,t^2$$
$$t^2 = 1{,}765$$

Taking the square-root we get t = +42 seconds or t = -42 seconds.
Since we are considering a time in the future of the present moment when h = 300 we eliminate the negative solution and get **t = +42 seconds**. This is just under 1 minute!



For over 100 years, astronomers have been investigating how interstellar dust absorbs and reflects starlight. Too much dust and stars fade out and become invisible to optical telescopes.

NASA's infrared observatories such as WISE and Spitzer, study dust grains directly through the infrared 'heat' radiation that they emit. The amount of heat radiation depends on the chemical composition of the dust grains and their reflectivity (called the albedo). Through detailed studies of the electromagnetic spectrum of dust grains, astronomers can determine their chemical composition.

The two images to the left, taken with the European Space Agency's Very Large Telescope, show the optical (top) and infrared (bottom) appearance of the interstellar dust cloud Barnard 68. They show how the dust grains behave at different wavelengths. At visible wavelengths, they make the cloud completely opaque so distant background stars cannot be seen at all. At infrared wavelengths, the dust grains absorb much less infrared light, and the cloud is nearly transparent. (Courtesy

$$A(m) = c\frac{\left|\dfrac{m^2-1}{m^2+2}\right|^2}{\mathrm{Im}\left(\dfrac{1-m^2}{m^2+2}\right)}$$

The equation above is a mathematical model of the albedo of a dust grain, A(m), as a function of its index of refraction, m, which is a complex number of the form m = a - bi. The denominator Im(…) is the 'imaginary part' of the indicated complex quantity in parenthesis. From your knowledge of complex numbers and their algebra, answer the questions below.

Problem 1 - An astronomer uses a dust grain composition of pure graphite for which m = 3 - i. What is the albedo of a 0.1 micron diameter dust grain at

A) Ultraviolet wavelengths of 0.3 microns (c= 10.0)?

B) At an infrared wavelength of 1 micron (c= 0.1)?

Problem 1 - An astronomer uses a dust grain composition of pure graphite for which m= 3 - i. What is the albedo of a 0.1 micron dust grain at A) Ultraviolet wavelength of 0.3 microns (c= 10.0); and B) At an infrared wavelength of 1 microns (c= 0.10)?

Answer: Recall that $|a+bi| = \sqrt{a^2+b^2}$, $m^2 = (a+bi)(a+bi)$ and that $\mathrm{Im}(a+bi) = b$ then for a=3.0 and b= -1.0 we have:

$$m^2 - 1 = (3-i)(3-i) - 1 = \left[(3)^2 + (i)^2(1)^2 - 2(3)(1)i\right] - 1 = 7 - 6i$$

$$m^2 + 2 = (3-i)(3-i) + 2 = \left[8 - 6i\right] + 2 = 10 - 6i$$

then $A(m) = c\dfrac{\left|\dfrac{7-6i}{10-6i}\right|^2}{\mathrm{Im}\left(\dfrac{6i-7}{10-6i}\right)}$

The complex fraction is evaluated by multiplying the numerator and denominator by the conjugate of 10 - 6i which is 10 + 6i so (10 - 6i) x (10 + 6i)= $10^2 + 36^2$ = 136. The numerator becomes (7 - 6i) x (10 + 6i) = 70 - 60i +42i -36(-1) = 106 - 18i.

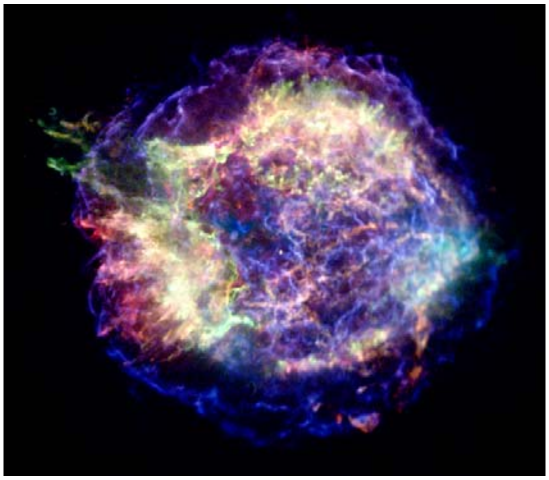
Then (106 - 18i)/136 = 0.78 - 0.18i. So we get:

$$A(m) = c\frac{|0.78 - 0.18i|^2}{\mathrm{Im}(-0.78 + 0.18i)} = c\frac{0.78^2 + 0.18^2}{0.18} = 3.6c$$

Then for
A) c=10 and we have an ultraviolet albedo of A(m) = 10 x 3.6 = **36** and

B) for the infrared c=0.1 and we have A(m) = 0.1x3.6 = **0.36**.

Note to teacher: This means that at ultraviolet wavelengths the dust grain reflects 36/0.36 = 1000 times more energy than at infrared wavelengths for a dust grain of this size.


The expanding supernova shell Cass-A as seen by the Chandra X-ray Observatory (Courtesy: NASA/Chandra)

When a star explodes as a supernova, a shock wave travels from the center of the explosion into interstellar space. As the shell of debris expands, it compresses the interstellar gas surrounding it to a higher density. The amount of compression can be modeled by the formula:

$$Y(x) = 0.67x^2 + 6x - 2.66$$

where x is the ratio of the gas density ahead of the shock front to the density behind the shock front.

Problem 1 – Use the Quadratic Formula to find the two roots for y(x).

Problem 2 – What is the vertex location for y(x)?

Problem 3 - What is the graph for y(x)?

Problem 4 – Only choices for x that are positive-definite over the set of Real numbers are permitted solutions for y(x). What root for y(x) is a permitted solution for y(x)?

Problem 5 – The value for x that satisfies y(x)=0 gives the ratio of the gas density behind the shock wave, to the density of the undisturbed interstellar medium. By what factor was the interstellar medium compressed as it passed through the shock wave?

Problem 1 – Use the Quadratic Formula to find the two roots for y(x).

$Y(x) = 0.67x^2 + 6x - 2.66$

Answer: $X = [-6 +/- (36 – 4(0.67)(-2.66))^{1/2}]/ 2(0.67)$
X = -4.47 +/- 4.90 so **x1 = +0.43** and **x2 = -9.38**

Problem 2 – What is the vertex location for y(x)?
Answer: The vertex is located at x = -b/2a and y = $-b^2/4a$, so x = -6/(4)(0.67) so **x = -4.47** and y = -(6)2/4(0.67) so **y = -13.4**

Problem 3 - What is the graph for y(x)? Answer: See below.

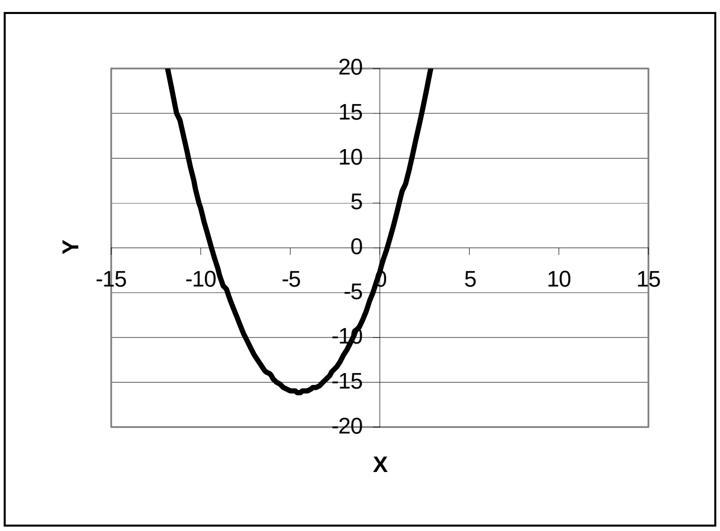
Problem 4 – Answer: The only permitted solution is **x = +0.43**

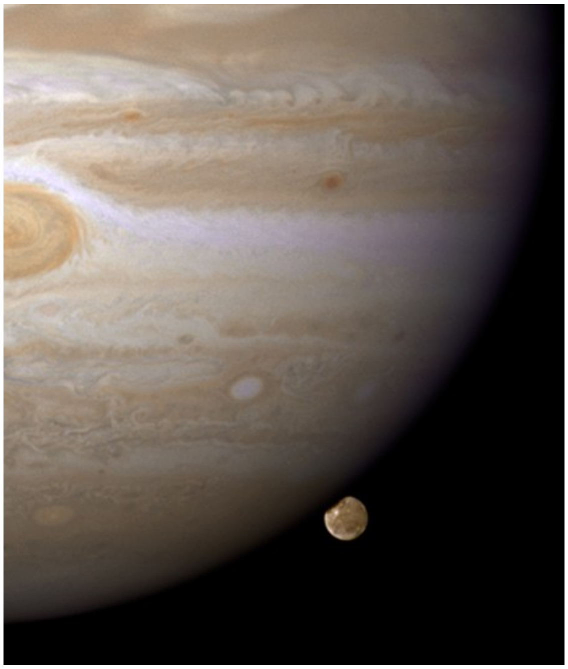
Problem 5 –

Answer: $X = \dfrac{\text{Gas density ahead of shock}}{\text{Gas density behind shock}} = +0.43$

So the inverse of x gives the desired compression of **2.32 times**.

Note: The formula for y(x) was derived by astronomers Shull and Draine, in the book 'Interstellar Processes' page 288-289, and represents the amount of entropy (disorder) created by the shock front. The roots of the equation represent compressions for which the entropy change is zero.





Ganymede passing behind Jupiter during an occultation: April 9, 2007 (NASA/Hubble)

Although total solar eclipses are dramatic, astronomers use another related phenomenon to determine the size and shape of distant asteroids.

When an asteroid passes in front of a star during an 'occultation' astronomers can accurately measure when the star fades out and brightens as the asteroid passes by. Observers on Earth's surface will see this occultation tack at different orientations, and by combining their timing data, the shape of the asteroid can be found.

The formula below shows the occultation track of a hypothetical star as the Moon passes-by.

$$D^2(t) = 225t^2 - 3825t + 16545$$

In this formula, t is the time in minutes, and D(t) is the distance between the center of the Moon and the center of the passing star in arcminutes. In the following problems adopt the definition that $Y(t) = D^2(t)$ to simplify the form of the equation.

Problem 1 – Use the Quadratic Formula to show that, for this particular star occultation, under no conditions will Y(t)=0.

Problem 2 – At what time, t, will the star reach its closest distance to the center of the Moon?

Problem 3 – What will be the distance, in arcminutes, between the star and the center of the Moon at its closest approach?

Problem 4 – If the radius of the Moon is 15 arcminutes, how close to the edge of the Moon will the star pass at its closest approach?

Problem 5 – Graph the function Y(t). At what times will the star be 41 arcminutes from the center of the Moon?

Problem 1 – $Y(t) = 225t^2 - 3825t + 16545$

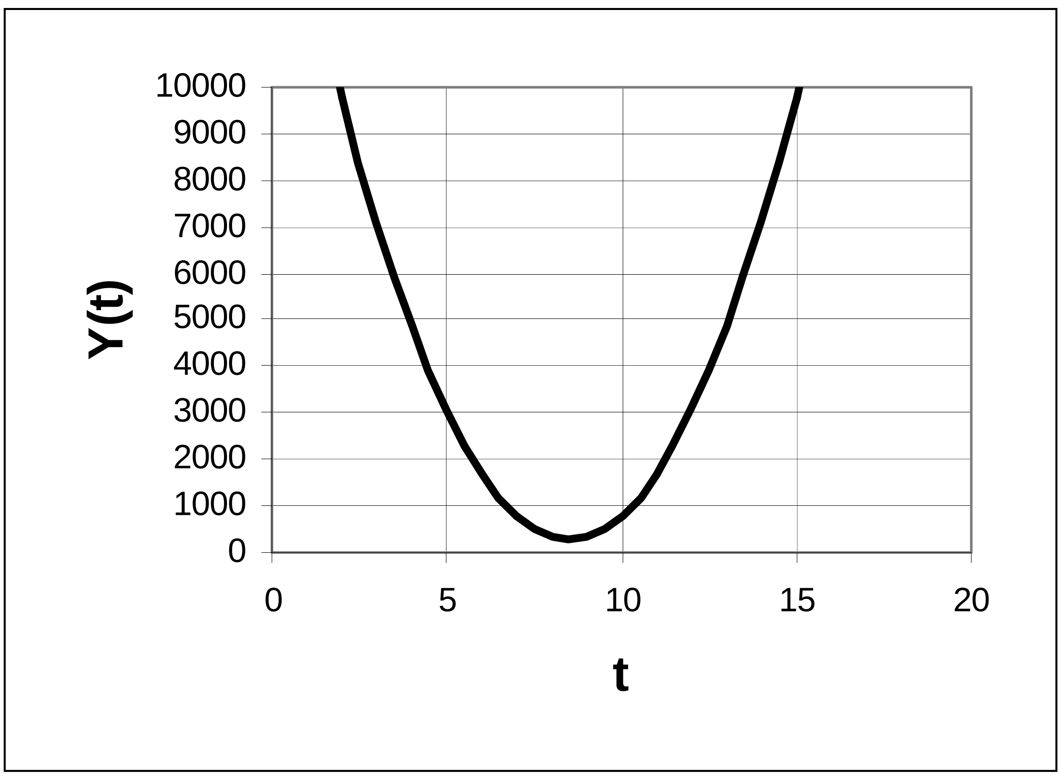
Answer: **The discriminant is $(-3825)^2 - 4(225)(16545) = -259875$ which is negative so the roots are imaginary and so there can be no times for which y=0.**

Problem 2 –Answer: This is equivalent to finding the location of the vertex (t,Y) of this parabola, since the parabola opens upwards and the vertex represents the minimum value for Y(t). Answer: $t = -b/2a = 3825/(2 \times 225)$ so **t = +8.5 minutes**

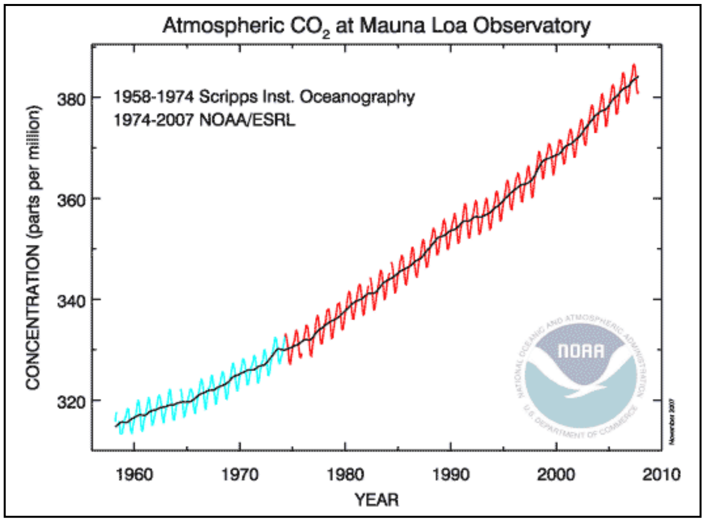
Problem 3 – Answer: First find the Y value of the vertex from by evaluating Y(t) at t = +8.5 : $Y = 225(8.5)^2 - 3825(8.5) + 16545 = +289$. Since the actual distance is defined as $D = Y^{1/2}$ we have $D = (289)^{1/2}$ = **17 arcminutes**.

Problem 4 – Answer: The closest approach of the star to the center of the Moon was 17 arcminutes, and since the Moon's radius is 15 arcminutes, the distance from the edge of the Moon is just 17 arcminutes – 15 arcminutes = **2 arcminutes**.

Problem 5 – Graph the function Y(t). At what times will the star be 41 arcminutes from the center of the Moon? Answer: See graph below. The values for $Y(t) = 41^2 = 1681$. These occur at **t = +6 minutes and t = +11 minutes** . This can be verified by solving for the roots of $1681 = 225t^2 - 3825t + 16545$ using the Quadratic Formula.



Modeling with quadratic functions



The annual change in carbon dioxide in the atmosphere, shown in the 'Keeling Curve' to the left, is a matter of grave concern since it contributes to global warming and climate change.

The data for the carbon dioxide rise since 1960 is shown in the table below. The 'Year' is the number of years since 1960 and 'CO2' is the concentration of carbon dioxide in the atmosphere in parts per million (PPM)

Year	0	10	20	30	40	50
CO2	317	326	338	354	369	390

Problem 1 – A portion of the Keeling Curve can be modeled as a parabolic equation in the form

$$CO2(t) = at^2 + bt + c$$

where t is the number of years from 1960. Using three data points in the table, solve a set of three simultaneous, linear equations to determine the best-fit quadratic equation for this atmospheric data.

Problem 2 – Graph the data and your best-fit quadratic equation for the period from 1960 to 2100 in decade intervals.

Prob**lem 3** - What is your prediction for the year A) 2020? B) 2050? C) 2100?

Problem 1 – Solve a set of three simultaneous, linear equations to determine the best-fit quadratic equation for this atmospheric data.

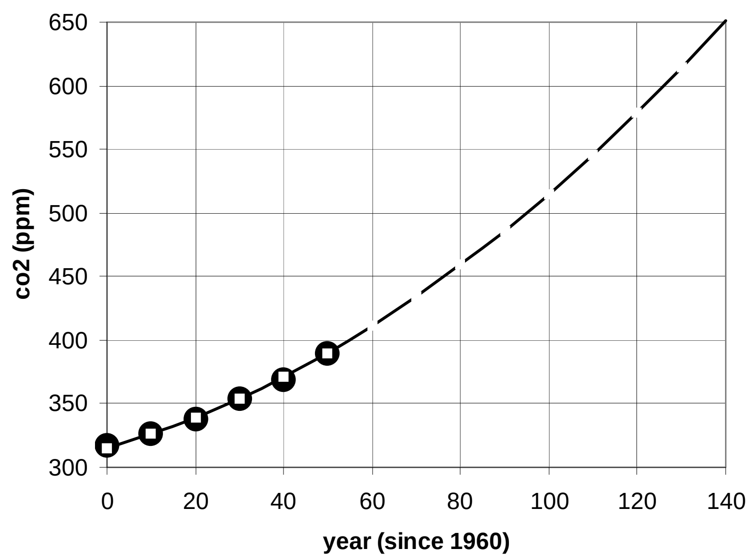
Answer: Select three points in the data, for example [10,326], [30,338] and [50,390]. The general form for a quadratic equation is $y = ax^2 + bx + c$ so the equation for each of the three points are:

$326 = a(10)^2 + b(10) + c$	$326 = 100a + 10b + c$
$354 = a(30)^2 + b(30) + c$	$354 = 900a + 30b + c$
$390 = a(50)^2 + b(50) + c$	$390 = 2500a + 50b + c$

The solution for this set of three equations can be found by any convenient method and leads to the solution a = 0.01 b= +1.0 c = 315

So $\mathbf{y = 0.01\, t^2 + 1.0\, t + 315}$

Problem 2 – Graph the data and your best-fit quadratic equation for the period from 1960 to 2100 in decade intervals over the range Y:[300, 600]



Prob**lem 3** - What is your prediction for the year A) 2020? B) 2050? C) 2100?
Answer: **A) 411 ppm, B) 486 ppm, C) 651 ppm.**

Note: More sophisticated climate models forecast 500 to 950 ppm by 2100 depending on the industrial response to this growth.

Students may also use online equation solvers such as http://www.akiti.ca/SimEq3Solver.html or http://mkaz.com/math/js_lalg3.html

Modeling with quadratic functions

The Crab Nebula is all that remains of a star that exploded as a supernova in the year 1054 AD. Astronomers have studied it carefully to investigate the causes of these explosions, and what happens to the left-over matter.

The NASA image seen here is a combination of optical data from Hubble Space Telescope (red areas) and X-Ray data from Chandra X-ray Observatory (blue areas).

The data for the energy emitted by this nebula is given in the table below. F is the logarithm of the frequency of the radiation in Hertz; E is the base-10 logarithm of the amount of energy emitted by the nebula in ergs/cm^2/sec:

F	7	10	13	16	19	22	25	28
E	-12.5	-11	-9	-7.5	-8.5	-10	-11	-13

Problem 1 – Solve a set of three simultaneous, linear equations to determine the best-fit quadratic equation for this energy data. Use the three points in the data at [10, -11], [16, -7.5] and [25,-11].

Problem 2 – Graph the data and your best-fit quadratic equation for the Log(frequency) domain from 5 to 30, over the Log(energy) range Y:[-16, -6]

Problem 3 - What is your prediction for the energy produced by the Crab Nebula in the radio region of the electromagnetic spectrum, at a frequency of 1 megaHertz (Log(F) = +6) ?

Problem 4 – At what frequency would you predict is the emission the largest in Log(E)?

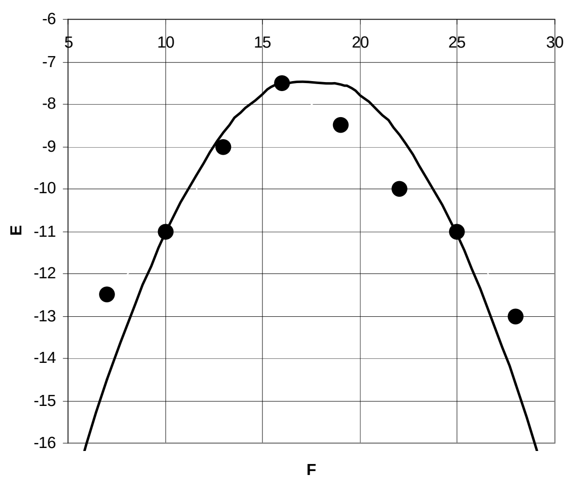
Problem 1 –Answer: The general form for a quadratic equation is $y = ax^2 + bx + c$ so the equation for each of the three points are:

$-11 = a(10)^2 + b(10) + c$ $-11 = 100a + 10b + c$
$-7.5 = a(16)^2 + b(16) + c$ $-7.5 = 256a + 16b + c$
$-11 = a(25)^2 + b(25) + c$ $-11 = 625a + 25b + c$

The solution for this set of three equations can be found by any convenient method and leads to the solution a = -0.065 b= +2.27 c = -27.2

So $\mathbf{E = -0.065\, F^2 + 2.27\, F - 27.5}$

Problem 2 – Graph the data and your best-fit quadratic equation for the Log(F) domain from 5 to 30, over the Log(E) range Y:[-16, -6]



Problem 3 - What is your prediction for the energy produced by the Crab Nebula in the radio region of the electromagnetic spectrum, at a frequency of 1 megaHertz (Log(F) = +6) ?

Answer: $E = -0.065\,(6)^2 + 2.27\,(6) - 27.5$ so **E = -16.2**

Problem 4 – At what frequency would you predict is the emission the largest in Log(E)?

Answer: Near F = +17.5 (Note, this corresponds to a frequency of $10^{+17.5} = 3.2 \times 10^{17}$ Hertz and is in the x-ray region of the electromagnetic spectrum.).

Students may also use online equation solvers such as http://www.akiti.ca/SimEq3Solver.html or http://mkaz.com/math/js_lalg3.html

Modeling with quadratic functions

The Comet Tempel-1 like many comets emits water molecules as it passes the sun and heats up.

This image taken by the Deep Impact spacecraft shows the nucleus of the comet, about 10 km across, and a brilliant plume of gas and dust ejected as the Deep Impact spacecraft flew by and launched a 370-kg impactor to strike the nucleus on July 4, 2005. The ejected material was studied to determine its composition.

The data for the quantity of water ejected by the comet as it approaches the Sun has been measured by astronomers. Time, T, is measured in the number of days after its closest passage to the Sun, and the amount of water, W, is measured in terms of kilograms of water emitted per second. The data table below gives the results:

T	-150	-100	-50	0	+50	+100	+150
W	72	116	174	203	174	145	87

Problem 1 – Solve a set of three simultaneous, linear equations to determine the best-fit quadratic equation for this water data. Use the three points in the data at [-100, 116], [0, 203] and [+100, 145].

Problem 2 – Graph the data and your best-fit quadratic equation for the time domain from -150 to +150 days, over the range W:[0, 250]

Problem 3 - What is your prediction for the water produced by Comet Tempel-1 at the time when the Deep Impact spacecraft flew past the comet at T = -1, just one day before the comet reached perihelion?

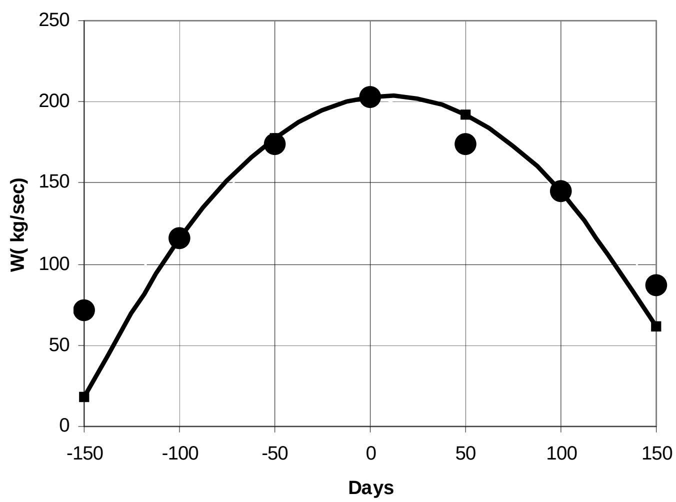
Problem 1 –Answer: The general form for a quadratic equation is $y = ax^2 + bx + c$ so the equation for each of the three points are:

$116 = a(-100)^2 + b(-100) + c$ $\quad\quad$ $116 = 10000a - 100b + c$
$203 = a(0)^2 + b(0) + c$ $\quad\quad$ $203 = c$
$145 = a(100)^2 + b(100) + c$ $\quad\quad$ $145 = 10000a + 100b + c$

The solution for this set of three equations can be found by any convenient method and leads to the solution a = -0.0073 b= +0.15 c = +203

So $\mathbf{W = -0.0073\ T^2 + 0.15\ T + 203}$

Problem 2 – Graph the data and your best-fit quadratic equation for the time domain from -150 to +150 days, over the range W:[0, 250]

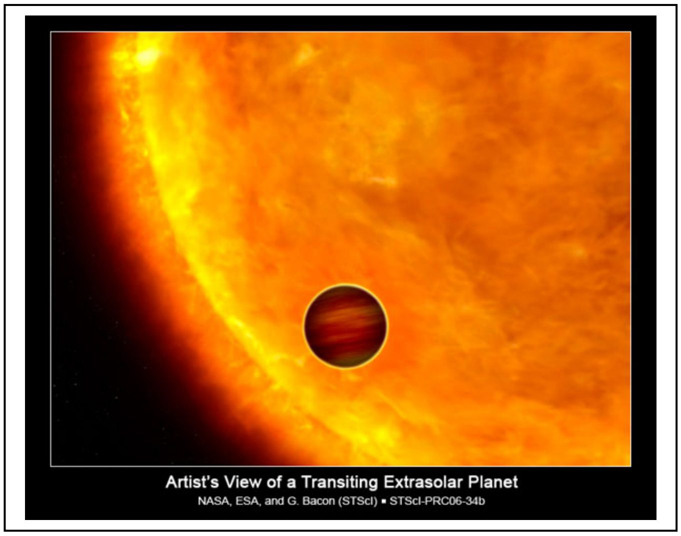


Problem 3 - What is your prediction for the water produced by Comet Tempel-1 at the time when the Deep Impact spacecraft flew past the comet at T = -1, just one day before the comet reached perihelion?

Answer: $W = -0.0073\ (-1)^2 + 0.15\ (-1) + 203$ so $\mathbf{W = 202.8\ kg/sec}$

Students may also use online equation solvers such as http://www.akiti.ca/SimEq3Solver.html or http://mkaz.com/math/js_lalg3.html

Modeling with quadratic functions



As technology improves, astronomers are discovering new planets orbiting nearby stars at an accelerating rate. These planets, called exoplanets so as not to confuse them with planets in our solar system, are often larger than our own planet Jupiter.

In addition, we can now detect planets that are only a few times more massive than Earth and may be Earth-like in other ways as well.

The number of exoplanets, N, discovered each year since 1996, Y, is approximately given in the data table below:

Y	0	2	4	6	8	10	12
N	5	8	20	30	45	55	70

Problem 1 – Solve a set of three simultaneous, linear equations to determine the best-fit quadratic equation for this exoplanet discovery data. Use the three points in the data at [0, 5], [6,30] and [12, 70].

Problem 2 – Graph the data and your best-fit quadratic equation for the time domain from 1996 to 2020, over the range W:[0, 200]

Problem 3 - What is your prediction for the number of planets that might be detected in the year 2014?

Answer Key

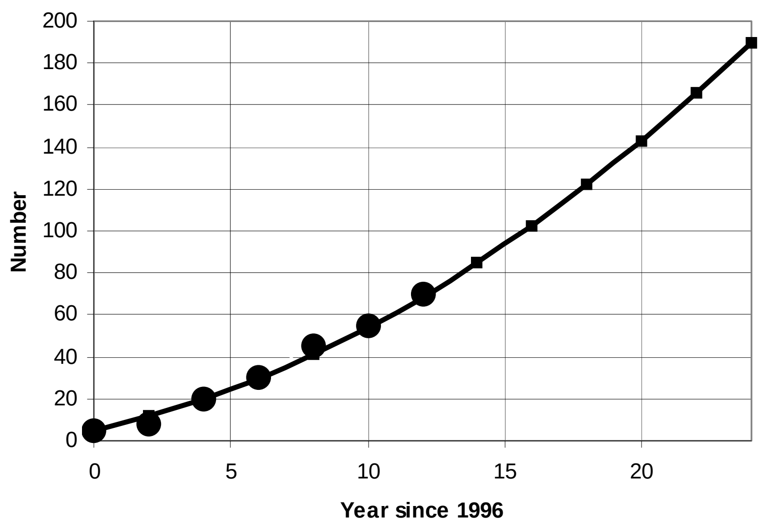
Problem 1 –Answer: The general form for a quadratic equation is $y = ax^2 + bx + c$ so the equation for each of the three points are:

$5 = a(0)^2 + b(0) + c$	$5 = c$
$30 = a(6)^2 + b(6) + c$	$30 = 36a + 6b + c$
$70 = a(12)^2 + b(12) + c$	$70 = 144a + 12b + c$

The solution for this set of three equations can be found by any convenient method and leads to the solution $a = +0.2$ $b = +2.9$ $c = +5$

So $\mathbf{N = 0.2\,Y^2 + 2.9\,Y + 5}$

Problem 2 – Graph the data and your best-fit quadratic equation for the time domain from 1996 to 2020 over the range N:[0, 20]



Problem 3 - What is your prediction for the number of planets that might be detected in the year 2014?

Answer: Y = 2014 – 1996 = 18
$N = 0.2\,(18)^2 + 2.9\,(18) + 5$ so **N = 122**

Students may also use online equation solvers such as
http://www.akiti.ca/SimEq3Solver.html or http://mkaz.com/math/js_lalg3.html

Note: The Kepler spacecraft has detected over 1200 new exoplanets since 2009 so the forecast above does not take into consideration the advent of new technology.