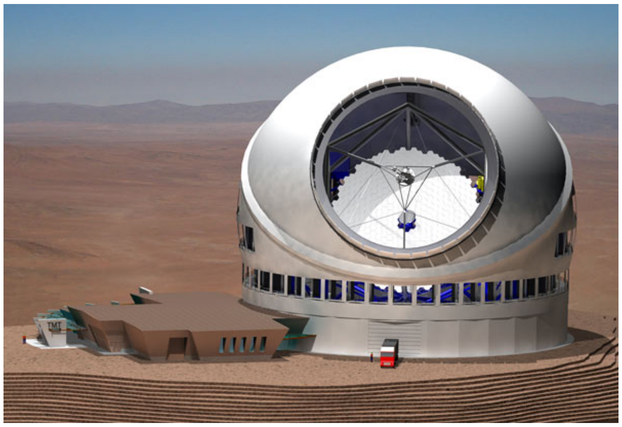


The size of a telescope mirror determines how well it can resolve details on distant objects. Astronomers are always building bigger telescopes to help them see the distant universe more clearly.

This artist's rendering shows the proposed Thirty-Meter Telescope mirror inside the observatory dome. Credit: TMT Observatory Corporation

Problem 1 - This simple function predicts the resolution, R(D) in angular seconds of arc, of a telescope mirror whose diameter, D, is given in centimeters:

$$R(D) = \frac{10.3}{D} \text{ arcseconds}$$

If the domain of R(D) extends from the size of a human eye of (0.5 centimeters), to the diameter of the Hubble Space Telescope (240 centimeters), what is the angular range of R(D) in arc seconds?

Problem 2 - Over what domain of the function R(D) will the resolution exceed 1 arcsecond?

Problem 3 - Fill in the missing numbers in the tabular form of R(D) shown below. Use two significant figure accuracy by rounding where appropriate:

D		1.0		20.0		100.0		200	
R(D)	21.0		2.1		0.21		0.069		0.043

Problem 1 - This simple function predicts the resolution, R(D) in angular seconds of arc, of a telescope mirror whose diameter, D, is given in centimeters:

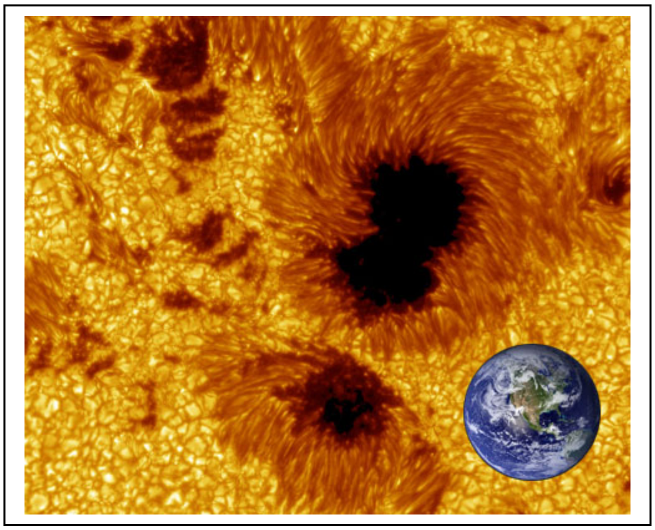
$$R(D) = \frac{10.3}{D}$$

If the domain of R(D) extends from the size of a human eye of (0.5 centimeters), to the diameter of the Hubble Space Telescope (240 centimeters), what is the angular range of R(D) in arc seconds?
Answer: R(0.5) = 21.0 arcseconds and R(240) = 0.043 arcseconds so the range is **[0.043, 21.0]**

Problem 2 - Over what domain of the function R(D) will the resolution exceed 1 arcsecond? Answer: For all values of D such that 1.0 > 10.3/D and so **D > 10.3 cm**.

Problem 3 - Fill in the missing numbers in the tabular form of R(D) shown below. Use two significant figure accuracy by rounding where appropriate:

D	**0.5**	1.0	**5.0**	20.0	**50.0**	100.0	**150.0**	200	**240**
R(D)	21.0	**10.0**	2.1	**0.52**	0.21	**0.10**	0.069	**0.052**	0.043



The sun goes through a periodic cycle of sunspots being common on its surface, then absent. Sunspot counts during the last 200 years have uncovered many interesting phenomena in the sun that can lead to hazardous 'solar storms' here on Earth.

During the most recent sunspot cycle, Number 23, the average annual number of spots, N, discovered each year, Y, was given by the table below:

Y	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
N	9	21	64	93	120	111	104	64	40	30	15

Problem 1 - Graph this data for N(Y).

Problem 2 - How do you know that the data represents a function rather than merely a relation?

Problem 3 - What is the domain and range of the sunspot data?

Problem 4 - When did the maximum and minimum occur, and what values did N(Y) attain? Express your answers in functional notation.

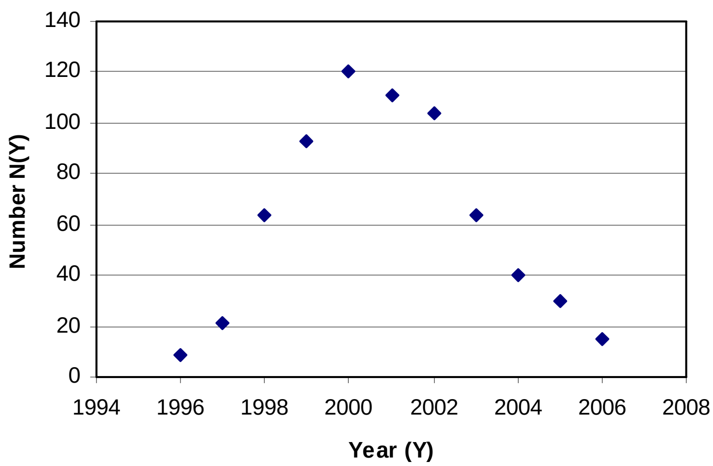
Problem 5 - Over what domain was the range below 50% of the maximum?

Problem 1 - During the most recent sunspot cycle, Number 23, the average annual number of spots, N, counted each year, Y, was given by the table below:

Y	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
N	9	21	64	93	120	111	104	64	40	30	15

Write this information in functional notation so that it is easier to refer to this information. Answer: N(Y)

Problem 1 - Graph this data.



Problem 2 - How do you know that the data represents a function rather than merely a relation? Answer: **From the vertical line test, every Y only touches one value of N(Y).**

Problem 3 - What is the domain and range of the sunspot data?
Answer: **Domain [1996, 2006] Range [9,120]**

Problem 4 - When did the maximum and minimum occur, and what values did N(Y) attain? Express your answers in functional notation.

Answer: The maximum occurred for **Y=2000 with a value of N(2000)=120 sunspots**; the minimum occurred for **Y=1996 with a value N(1996) = 9 sunspots**.

Problem 5 - Over what domain was the range below 50% of the maximum?

Answer: The maximum was 120 so 1/2 the maximum is 60. N(Y) < 60 occurred for **[1996,1997] and [2004,2006]**.

Slope and Rate of Change

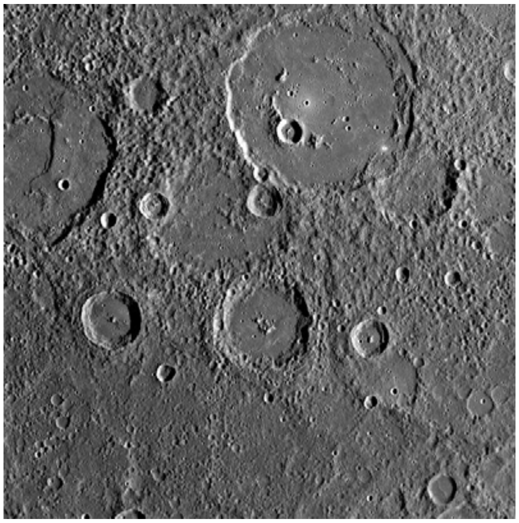


Image of craters on Mercury taken by the MESSENGER spacecraft.

Because things change in the universe, astronomers often have to work with mathematical quantities that describe complex rates.

Definition: A rate is the ratio of two quantities with different units.

In the problems below, convert the indicated quantities into a rate.

Example: 15 solar storms in 2 weeks becomes the rate:

$$R = \frac{15 \text{ solar storms}}{2 \text{ weeks}} = \frac{15}{2}$$

R = 7 solar storms/week.
or 7 solar storms per week.

Problem 1 - 15 meteor impacts in 3 months.
Problem 2 - 2,555 days in 7 years
Problem 3 - 1,000 atomic collisions in 10 seconds
Problem 4 - 36 galaxies in 2 two clusters
Problem 5 - 1600 novas in 800 years
Problem 6 - 416 gamma-ray bursts spotted in 52 weeks
Problem 7 - 3000 kilometers traveled in 200 hours.
Problem 8 - 320 planets orbiting 160 stars.
Problem 9 - 30 Joules of energy consumed in 2 seconds

Compound Units:

Problem 10 - 240 craters covering 8 square kilometers of area
Problem 11 - 16,000 watts of energy collected over 16 square meters.
Problem 12 - 380 kilograms in a volume of 20 cubic meters
Problem 13 - 6 million years for 30 magnetic reversals
Problem 14 - 1,820 Joules over 20 square meters of area
Problem 15 - A speed change of 50 kilometers/sec in 10 seconds.

Scientific Notation:

Problem 16 - 3×10^{13} kilometers traveled in 3×10^{7} seconds.
Problem 17 - 70,000 tons of gas accumulated over 20 million square kilometers
Problem 18 - 360,000 Newtons of force over an area of 1.2×10^{6} square meters
Problem 19 - 1.5×10^{8} kilometers traveled in 50 hours
Problem 20 - 4.5×10^{5} stars in a cluster with a volume of 1.5×10^{3} cubic lightyears

Answer Key

Problem 1 - 15 meteor impacts in 3 months. **= 5 meteor impacts/month**.
Problem 2 - 2,555 days in 7 years = 2,555 days / 7 years = **365 days/year**
Problem 3 - 1,000 atomic collisions in 10 seconds = **100 atomic collisions/second**
Problem 4 - 36 galaxies in 2 two clusters = **18 galaxies/cluster**
Problem 5 - 1600 novas in 800 years = **2 novas/year**
Problem 6 - 416 gamma-ray bursts spotted in 52 weeks = **8 gamma-ray bursts/week**
Problem 7 - 3000 kilometers traveled in 200 hours. = **15 kilometers/hour**
Problem 8 - 320 planets orbiting 160 stars. = **2 planets/star**
Problem 9 - 30 Joules of energy consumed in 2 seconds = **15 Joules/second**

Compound Units:

Problem 10 - 240 craters covering 8 square kilometers of area = **30 craters/km^2**
Problem 11 - 16,000 watts of energy collected over 16 square meters. = **1000 watts/m^2**
Problem 12 - 380 kilograms in a volume of 30 cubic meters = **19 kilograms/m^3**
Problem 13 - 6 million years for 30 magnetic reversals = **200,000 years/reversal**
Problem 14 - 1,820 Joules over 20 square meters of area = **91 Joules/m^2**
Problem 15 - A speed change of 50 kilometers/sec in 10 seconds. = **5 km/sec^2**

Scientific Notation:

Problem 16 - 3×10^{13} kilometers traveled in 3×10^{7} seconds.
= **$1.0\text{x}10^{6}$ kilometers/sec**
Problem 17 - 70,000 tons of gas accumulated over 20 million square kilometers
= 70,000 tons/20 million km^2 = **0.0035 tons/km^2**
Problem 18 - 360,000 Newtons of force over an area of 1.2×10^{6} square meters
= 360,000 Newtons/1,200,000 m^2 = **0.3 Newtons/m^2**
Problem 19 - 1.5×10^{8} kilometers traveled in 50 hours
= 1.5×10^{8} km/50 hrs = **3 million km/hr**
Problem 20 - 4.5×10^{5} stars in a cluster with a volume of 1.5×10^{3} cubic lightyears
= **300 stars/cubic lightyear**

Period	Age (years)	Days per year	Hours per day
Current	0	365	
Upper Cretaceous	70 million	370	
Upper Triassic	220 million	372	
Pennsylvanian	290 million	383	
Mississippian	340 million	398	
Upper Devonian	380 million	399	
Middle Devonian	395 million	405	
Lower Devonian	410 million	410	
Upper Silurian	420 million	400	
Middle Silurian	430 million	413	
Lower Silurian	440 million	421	
Upper Ordovician	450 million	414	
Middle Cambrian	510 million	424	
Ediacarin	600 million	417	
Cryogenian	900 million	486	

We learn that an 'Earth Day' is 24 hours long, and that more precisely it is 23 hours 56 minutes and 4 seconds long. But this hasn't always been the case. Detailed studies of fossil shells, and the banded deposits in certain sandstones, reveal a much different length of day in past eras! These bands in sedimentation and shell-growth follow the lunar month and have individual bands representing the number of days in a lunar month. By counting the number of bands, geologists can work out the number of days in a year, and from this the number of hours in a day when the shell was grown, or the deposits put down. The table above shows the results of one of these studies.

Problem 1 - Complete the table by calculating the number of hours in a day during the various geological eras in decimal form to the nearest tenth of an hour. It is assumed that Earth orbits the sun at a fixed orbital period, based on astronomical models that support this assumption.

Problem 2 - Plot the number of hours lost compared to the modern '24 hours' value, versus the number of years before the current era.

Problem 3 - By finding the slope of a straight line that best passes through the distribution of points in the graph, can you estimate by how much the length of the day has increased in seconds per century?

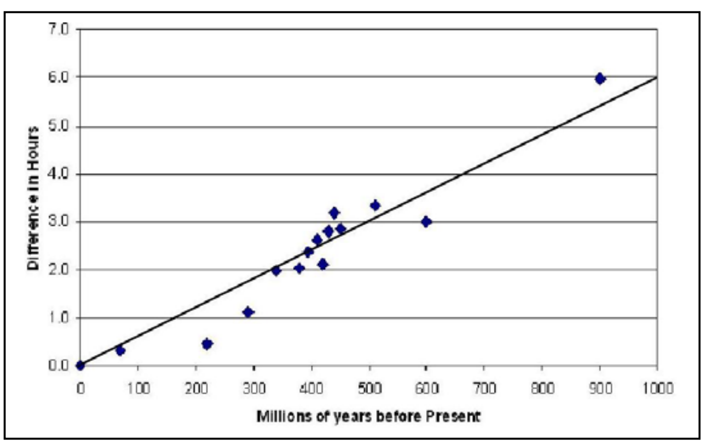
Answer Key

Period	Age (years)	Days per year	Hours per day
Current	0	365	**24.0**
Upper Cretaceous	70 million	370	**23.7**
Upper Triassic	220 million	372	**23.5**
Pennsylvanian	290 million	383	**22.9**
Mississippian	340 million	398	**22.0**
Upper Devonian	380 million	399	**22.0**
Middle Devonian	395 million	405	**21.6**
Lower Devonian	410 million	410	**21.4**
Upper Silurian	420 million	400	**21.9**
Middle Silurian	430 million	413	**21.2**
Lower Silurian	440 million	421	**20.8**
Upper Ordovician	450 million	414	**21.2**
Middle Cambrian	510 million	424	**20.7**
Ediacarin	600 million	417	**21.0**
Cryogenian	900 million	486	**18.0**

Problem 1 - Answer; **See table above. Example for last entry: 486 days implies 24 hours x (365/486) = 18.0 hours in a day.**

Problem 2 - Answer; **See figure below**

Problem 3 - Answer: From the line indicated in the figure below, the slope of this line is m = (y2 - y1)/ (x2 - x1) = 6 hours / 900 million years or 0.0067 hours/million years. Since there are 3,600 seconds/ hour and 10,000 centuries in 1 million years (Myr), this unit conversion yields 0.0067 hr/Myr x (3600 sec/hr) x (1 Myr / 10,000 centuries) = **0.0024 seconds/century.** This is normally cited as 2.4 milliseconds per century.



Slope-Intercept Graphing: The Moon's Orbit

The moon is slowly pulling away from Earth. In the distant future, it will be much farther away from us than it is now. It is currently moving away at a rate of 3.8 centimeters per year. The following formula predicts the distance of the moon for a period extending up to 2 billion years from now:

$$D(T) = 38\,T + 385{,}000$$

where T is the elapsed time from today in millions of years, and D is the distance in kilometers

Problem 1 - Graph the function D(T) over the domain T:[0.0 , 2,000].

Problem 2 - What is the slope of the function?

Problem 3 - What is the Y-intercept for the function?

Problem 4 - What is the range of D(T) for the given domain?

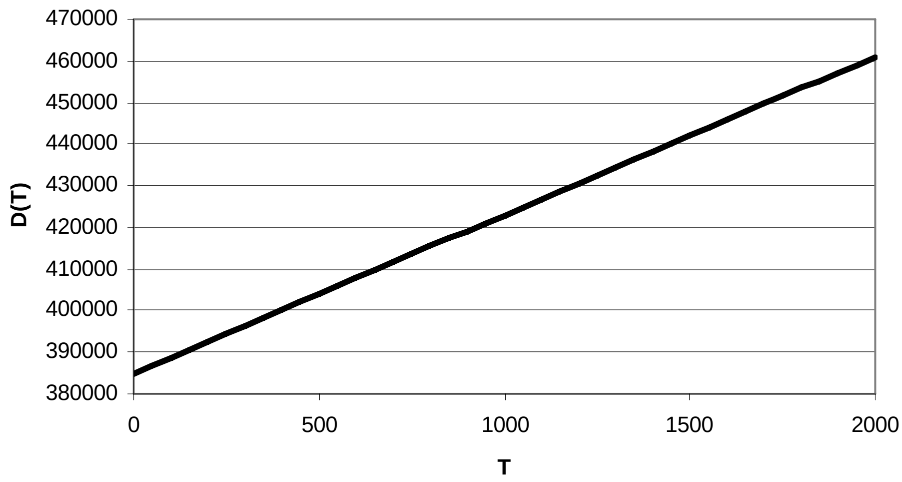
Problem 5 - How many years in the future will the orbit be exactly D(T) = 423,000 kilometers?

Problem 6 - How far from Earth will the Moon be by 500 million years from now?

Problem 7 - How far will the moon be from Earth by T = 3,000?

Answer Key

Problem 1 - Graph the function D(T) over the domain T:[0.0 , 2,000].



Problem 2 - What is the slope of the function?
Answer: From the equation, which is of the form y = mx + b, the slope
M= 38 kilometers per million years.

Problem 3 - What is the Y-intercept for the function? Answer: For T = 0, the y-intercept, D(0) = **385,000 kilometers**.

Problem 4 - What is the range of D(T) for the given domain?
Answer: For the domain T:[0,2000] , D(0) = 385,000 km and D(2000) = 461,000 km so the range is **D:[385,000 , 461,000]**

Problem 5 - How many years in the future will the orbit be exactly D(T) = 423,000 kilometers? Answer: solve 423,000 = 38T + 385,000

38T = 38,000

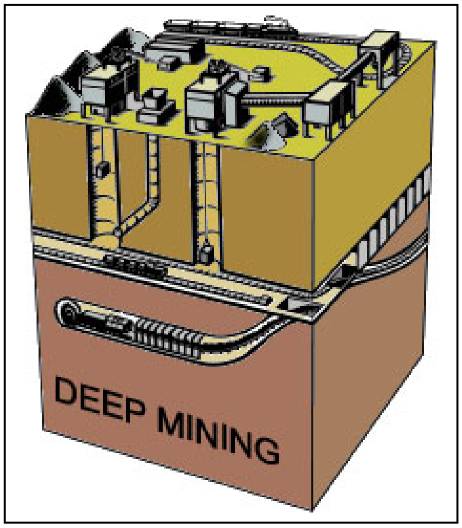
So T = 1,000

Since T is in units of millions of years, T = 1,000 is 1,000 million years or **1 billion years.**

Problem 6 - How far from Earth will the Moon be by 500 million years from now?
Answer: T = 500, so D(500) = 38(500) + 385,000 = **404,000 kilometers.**

Problem 7 - How far will the Moon be from Earth by T = 3,000?
Answer: **This value for T falls outside the stated domain of D(T) so we cannot use the function to determine an answer.**

Slope-Intercept Graphing: Hot times below!



It is a very uncomfortable job working in a deep mine. As you dig deeper into the Earth, the temperature of the rock around you increases. Near Earth's surface, this rate is about 0.013 degrees Celsius per meter. The average temperature, T, in Celsius, at a particular depth, d, in meters, is given by the formula:

$$T(d) = 0.013\ d + 12$$

Problem 1 - Graph the function T(d) over the domain T:[0.0 , 4,000].

Problem 2 - What is the slope of the function?

Problem 3 - What is the Y-intercept for the function?

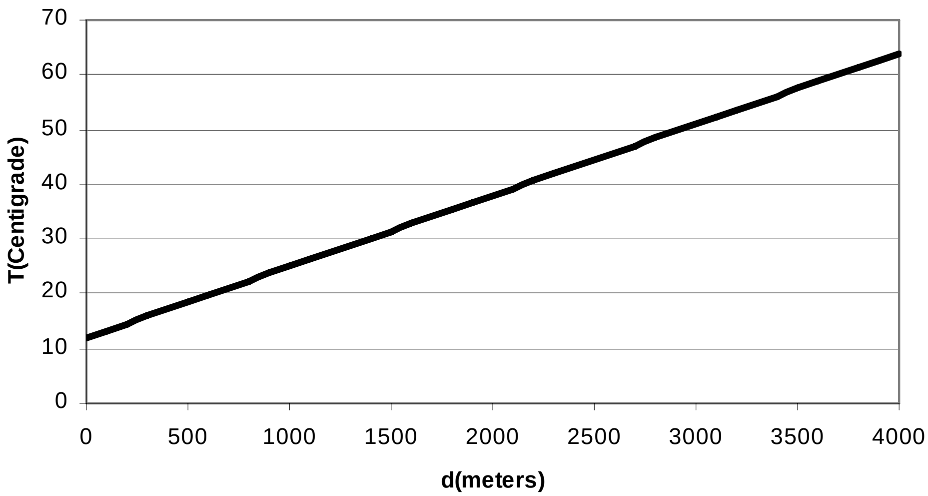
Problem 4 - What is the range of T(d) for the given domain?

Problem 5 - How far below the surface will the temperature be $T(d) = 140^{o}$ C?

Problem 6 - Water boils at 100^{o} C. How far down would you have to drill a well to reach temperatures where water boils?

Problem 7 - The deepest mine is a gold mine in South Africa. In 1977 the Western Deep Levels reached a depth of 3,581 meters. What would you predict is the temperature of the rocks at this depth?

Problem 1 - Graph the function T(d) over the domain T:[0.0 , 4,000].



Problem 2 - What is the slope of the function?
Answer; m = **0.013^{o}C per meter**.

Problem 3 - What is the Y-intercept for the function?
Answer: T(0) =**+12^{o} C**.

Problem 4 - What is the range of T(d) for the given domain?
Answer: T(0) = + 12^{o}C and T(4,000) = 0.013(4000) + 12 = +64^{o}C so the range is **T:[+12, +64]**

Problem 5 - How far below the surface will the temperature be T(d) = 140^{o}C?
Answer: 140 = 12 + 0.013d
128 = 0.013 d
So **d = 9,850 meters or 9.85 kilometers.**

Problem 6 - Water boils at 100^{o}C. How far down would you have to drill a well to reach temperatures where water boils?
Answer: 100 = 12 + 0.013d
88 = 0.013 d
So **d = 6,769 meters or 6.8 kilometers.**

Problem 7 - The deepest mine is a gold mine in South Africa; in 1977 the Western Deep Levels reached a depth of 3,581 meters. What would you predict is the temperature of the rocks at this depth?
Answer: T(3581) = 0.013 (3581) + 12 = **+58^{o}C**.

Note: This equals 136^{o}F! Deep mining requires cooling equipment to prevent miners suffering from heat stroke.

Slope-Intercept Graphing: Solar Power

Solar power can be a good thing, but even a slight change over millions of years can cause serious climate change. A 1% increase is enough to raise average Earth temperatures by 10 degrees Celsius. This will eventually spell the end of life on Earth billions of years from now.

Since it was first formed 4.5 billion years ago, our sun has steadily increased its brightness over time, and will continue to do so for billions of years to come. This will have important consequences for the continuation of life on Earth.

The equation that gives the sun's power output at Earth, P, over time, T, is given by:

$$P(T) = 1357 + 90\ T$$

where T is the time since today in billions of years, and P(T) is the amount of solar power, in watts, falling on each square meter of Earth's surface.

Problem 1 - Graph the function P(T) over the domain T:[-4.5 , +4.5]. What is the physical interpretation of this domain in time?

Problem 2 - What is the slope of the function including its units?

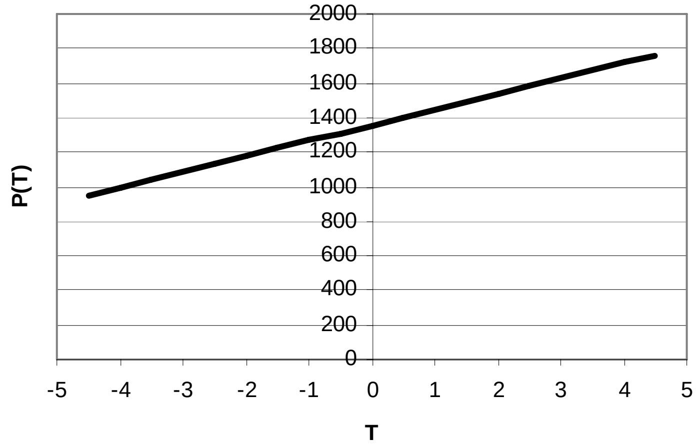
Problem 3 - What is the Y-intercept for the function including its units?

Problem 4 - What is the range of P(T) for the given domain including its units?

Problem 5 - As a percentage of its current solar power, what was the solar power at Earth's surface A) 500 million years ago? and; B) what will it be 500 million years from now?

Answer Key

Problem 1 - Graph the function P(T) over the domain T:[-4.5 , +4.5]. What is the interpretation of this range in time? Answer: The domain spans a time interval from 4.5 billion years ago, to 4.5 billion years into the future.



Problem 2 - What is the slope of the function including its units? Answer: **The slope is +90 watts per billion years**

Problem 3 - What is the Y-intercept for the function including its units? Answer: The 'y-intercept' is at **+1357 watts of solar power**.

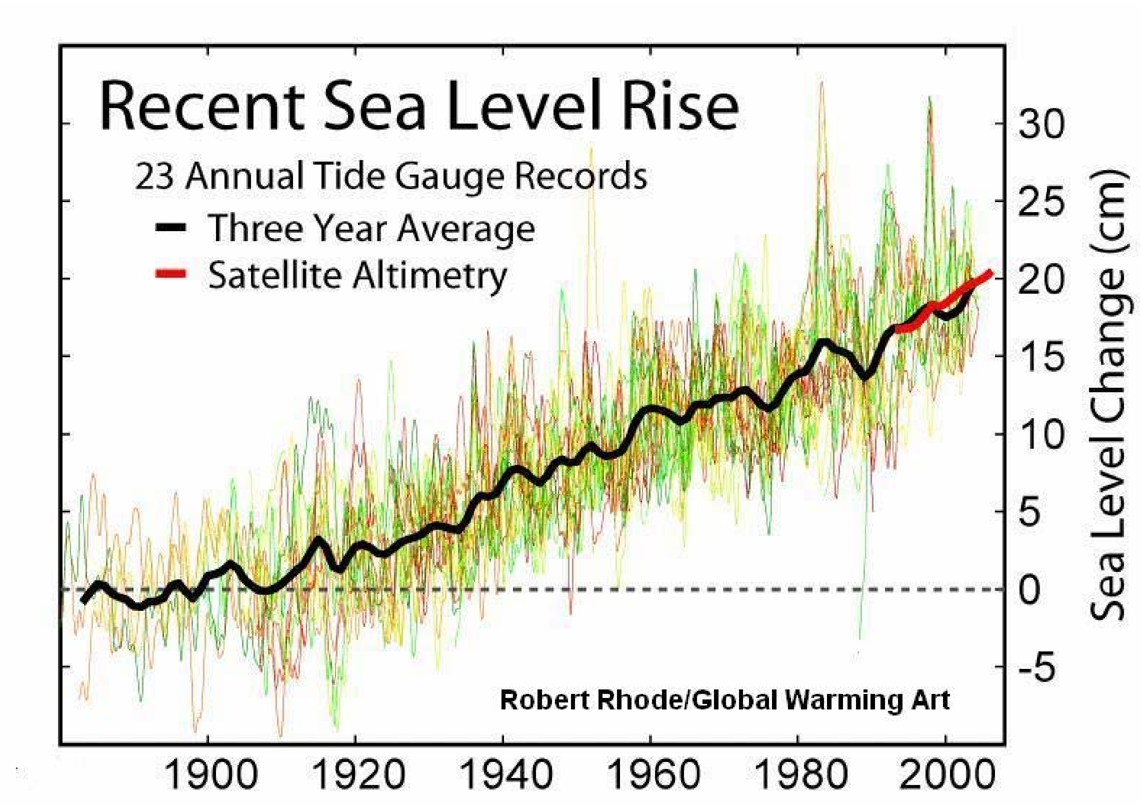
Problem 4 - What is the range of P(T) for the given domain including its units?
Answer: For T = -4.5 we have P(-4.5) = +952 watts. For T = +4.5 we have P(+4.5) = +1762 watts, so the range is **T:[+952, +1762]**.

Problem 5 - As a percentage of its current solar power, what was the solar power at Earth's surface A) 500 million years ago? and; B) what will it be 500 million years from now? Answer:

A) T(-0.5) = +1312 watts. Since at T(0) it is 1357 watts, the percentage is 100% x (1312/1357) = **97%**;

B) T(+0.5) = +1402 watts. Since at T(0) it is 1357 watts, the percentage is 100% x (1402/1357) = **103%**;

Writing Equations of Lines



The graph, produced by scientists at the University of Colorado and published in the IPCC Report-2001, shows the most recent global change in sea level since 1880 based on a variety of tide records and satellite data. The many colored curves show the individual tide gauge trends. The black line represents an average of the data in each year.

Problem 1 - If you were to draw a straight line through the curve between 1920 and 2000 representing the average of the data, what would be the slope of that line?

Problem 2 - What would be the equation of the straight line in A) Two-Point Form? B) Point-Slope Form? C) Slope-Intercept Form?

Problem 3 - If the causes for the rise remained the same, what would you predict for the sea level rise in A) 2050? B) 2100? C) 2150?

Problem 1 - If you were to draw a straight line through the curve between 1920 and 2000 representing the average of the data, what would be the slope of that line? Answer; See figure below. First, selecting any two convenient points on this line, for example X= 1910 and Y = 0 cm (1910, +0) and X = 1980 Y= +15 cm (1980, +15). The slope is given by m = (y2-y1)/(x2-x1) = 15 cm / 70 years = **0.21 cm/year.**

Problem 2 - What would be the equation of the straight line in A) Two-Point Form? B) Point-Slope Form? C) Slope-Intercept Form? Answer:

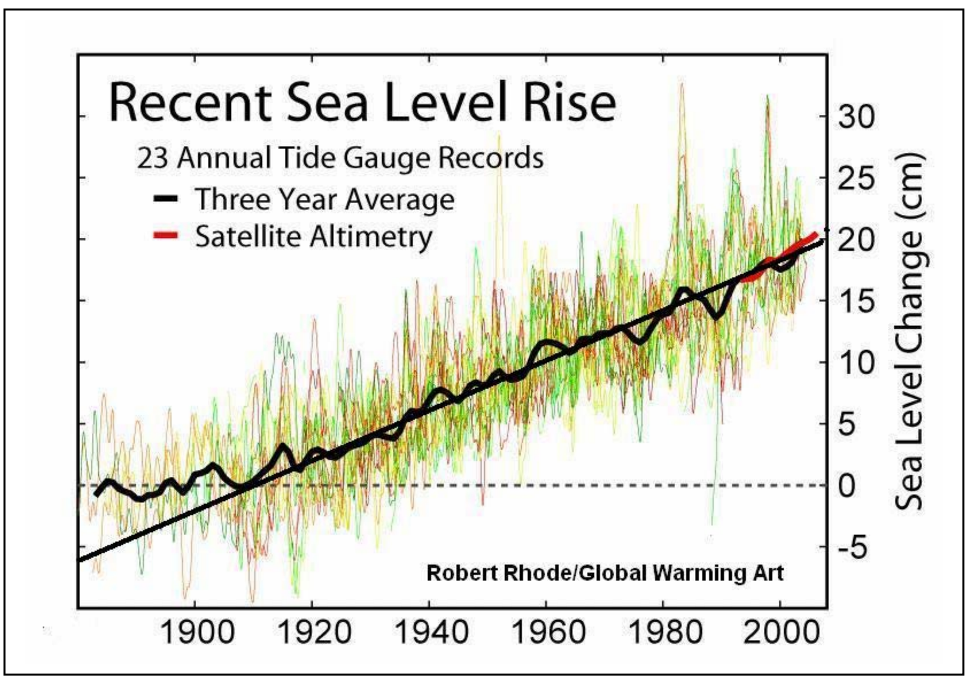
A) $y - y1 = \frac{(y2 - y1)}{(x2 - x1)} (x - x1)$ so $\mathbf{y - 0 = \frac{(15 - 0)}{(1980 - 1910)} (x - 1910)}$

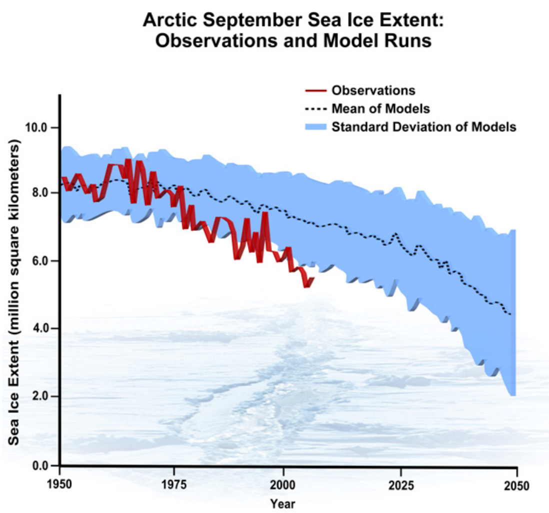
B) y - y1 = m (x - x1) so **y - 0 = 0.21 (x - 1910)**

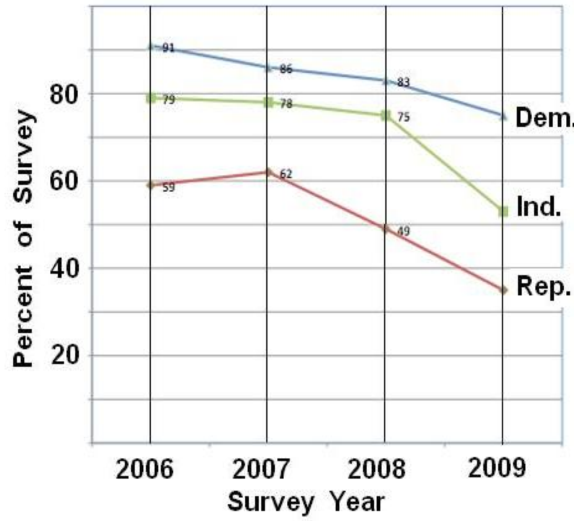
C) y = 0.21 x - 0.21(1910) so **y = 0.21x - 401.1**

Problem 3 - If the causes for the rise remained the same, what would you predict for the sea level rise in A) 2050? B) 2100? C) 2150? Answer:

A) y = 0.21 (2050) - 401.1 = **29.4 centimeters.** (Note; this equals 12 inches)
B) y = 0.21 (2100) - 401.1 = **39.9 centimeters** (Note: this equals 16 inches)
C) y = 0.21 (2150) - 401.1 = **50.4 centimeters.** (Note: this equals 20 inches)







The graph above, based upon research by the National Sea Ice Data Center (Courtesy Steve Deyo, UCAR), shows the amount of Arctic sea ice in September for the years 1950-2006, based on satellite data (since 1979) and a variety of direct submarine measurements (1950 - 1978). The blue region indicates model forecasts based on climate models. Meanwhile, the figure on the right shows the results of polls conducted between 2006 and 2009 of 1,500 adults by the Pew Research Center for the People & the Press. The graph indicates the percentage of people, in both major political parties and Independents, believing there is strong scientific evidence that the Earth has gotten warmer over the past few decades.

Problem 1 - Based on the red curve in the sea ice graph, which gives the number of millions of square kilometers of Arctic sea ice identified between 1950 and 2006, what is a linear equation that models the average trend in the data between 1950-2006?

Problem 2 - Based on the polling data, what are the three linear equations that model the percentage of Democrats (Dem.), Independents (Ind.) and Republicans (Rep.) who believed that strong evidence existed for global warming?

Problem 3 - From your linear model for Arctic ice cover, about what year will the Arctic Ice Cap have lost half the sea ice that it had in 1950-1975?

Problem 4 - From your model for the polling data, by about what years will the average American in the Pew Survey, who identifies themselves as Democrats, Independents or Republicans, no longer believe that there is any scientific evidence at all for global warming?

Problem 1 - Based on the red curve in the graph, which gives the number of millions of square kilometers of Arctic sea ice identified between 1950 and 2006, what is a linear equation that models the average trend in the data between 1950-2006? Answer: The linear equation will be of the form y = mx + b. From the graph, the y-intercept for the actual data is 8.5 million km^2 for 1950. The value for 2006 is 5.5 million km^2. The slope is m = (5.5 - 8.5) / (2006 - 1950) = -0.053, so the equation is given by **Y = -0.053(x-1950) + 8.5** in millions of km^2.

Problem 2 - Based on the polling data, what are the three linear equations that model the percentage of Democrats (Dem.), Independents (Ind.) and Republicans (Rep.) who believed that strong evidence existed for global warming?
Answer:
Dems: m = (75% - 90%)/(2009-2006) = -5.0, so the model becomes
y = -5.0(x - 2006) + 90 percent ;
Ind. m = (52% - 79%)/(2009-2006) = -9.0, so the model becomes
y = -9.0(x - 2006) + 79 percent.
Rep. ; (35% - 60%)/(2009-2006) = -8.3, so the model becomes
y = -8.3(x - 2006) + 60 percent

Problem 3 - From your linear model for Arctic ice cover, about what year will the Arctic Ice Cap have lost half the sea ice that it had in 1950-1975?
Answer: In 1950-1975 there were about 8.5 million km2 of sea ice in September. Half of this is 4.3 million km2. Set y = 4.3 and solve for x:

Solve 4.3 = -0.053(x-1950)+ 8.5 to get
-4.2 = -0.053(x-1950)
4.2 = 0.053(x-1950)
4.2/0.053 = x-1950
79 = x - 1950
And so x = 2029. So, during the year **2029 AD** there will only be half as much sea Ice in the Arctic in September.

Note: If we use only the slope data since 1975 when the ice cover was 8.0 million km^2, the slope would be m = (5.5 - 8.0)/(2006-1975) = -0.083, and linear equation is y = - 0.083(x-1975) + 8.0. The year when half the ice is present would then be about 2023 AD, because the slope is steeper during the most resent 30 years. If the slope continues to steepen with time, the year when only half the ice is present will move closer to the current year.

Problem 4 - From your model for the polling data, by about what years will the Democrats, Independents and Republicans no longer believe that there is any scientific evidence at all for global warming?
Answer: Solve each linear model in Problem 2 for X, given that y=0:
Democrats: 0 = -5.0(x - 2006) + 90 so x = **2024 AD**.
Independents: 0 = -9.0 (x-2006) + 79 so x = **2015 AD**
Republicans: 0 = - 8.3(x-2006) + 60 so x = **2013 AD**.

Correlation and Best-Fitting Lines

Time (sec)	Log(Brightness) (erg/sec/cm^2)
200	-10.3
500	-10.7
1,000	-11.0
6,000	-11.7
10,000	-12.0
25,000	-12.3
100,000	-13.0
500,000	-13.8

Gamma-ray bursts, first spotted in the 1960's, occur about once every day, and are believed to be the dying explosions from massive stars being swallowed whole by black holes that form in their cores, hours before the explosion. The amount of energy released is greater than entire galaxies of starlight.

This burst began January 16, 2005, and lasted 529,000 seconds as seen by the Swift satellite's X-ray telescope. The data for GRB 060116 is given in the table to the left. This source, located in the constellation Orion, but is over 10 billion light years behind the Orion Nebula!

Problem 1 - Plot the tabulated data on a graph with x = Log(seconds) and y = Log(Brightness).

Problem 2 - What is the best-fit linear equation that characterizes the data over the domain x: [2.0, 5.0]?

Problem 3 - What is the equivalent power-law function that represents the linear fit to the data?

Problem 4 - If the gamma-ray burst continues to decline at this rate, what will be the brightness of the source by A) February 16, 2005? B) January 16, 2006?

Problem 1 - Answer: See figure below.

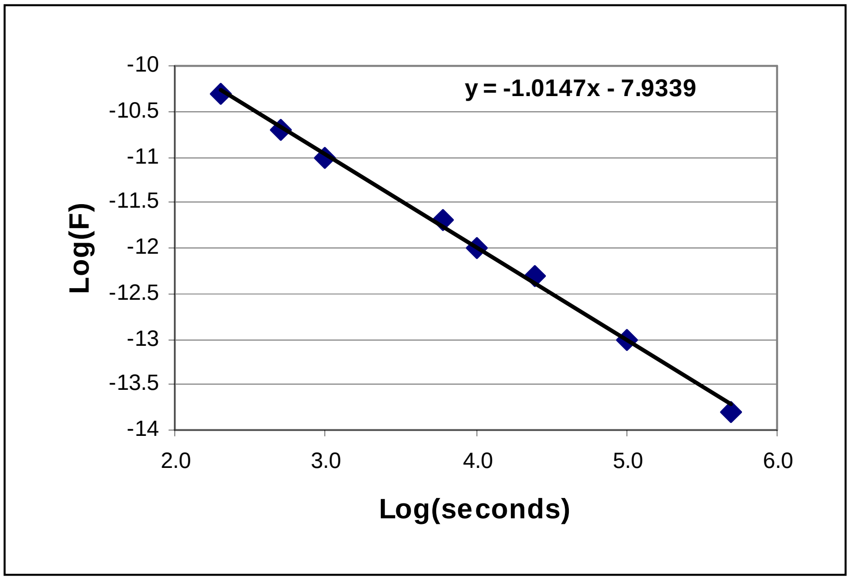
Problem 2 - Answer: See figure below with **y = -1.0x - 7.93**

Problem 3 - Answer:
LogB = -1.0Logt - 7.93 so
10LogB = 10(-1.0Logt - 7.93) or
$B(t) = 1.2\times10^{-8}\ t^{-1.0}$

Problem 4 - Answer:

A) First calculate the number of seconds elapsed between January 16 and February 16 which equals 31 days or 31 x (24 hrs) x (3600 sec/hr) = 2,678,400. Then
$B(t) = 1.17\times10^{-8}\ (2678400)^{-1.0}$ and so
$B(t) = 4.4 \times 10^{-15}$ ergs/sec/cm^2.

B) The elapsed time is 365 days or 3.1×10^{7} seconds so
$B(t) = 1.17\times10^{-8}\ (3.1\times10^{7})^{-1.0}$ and so
$B(t) = 3.8 \times 10^{-16}$ ergs/sec/cm^2.



Linear Inequalities in 2 Variables: A Star Cluster

Star clusters, like the one shown to the left, consist of hundreds of stars moving through space as a single unit.

Astronomers need to know the masses of these clusters, along with the numbers of the different types of stars that comprise them, in order to study how star clusters are formed and change in time.

The star cluster NGC 290 shown in this Hubble Space Telescope photo, is located in the nearby galaxy called the Small Magellanic Cloud about 200,000 light years from Earth.

Astronomers use the mass of our sun as a convenient unit of mass when comparing other stars. '1 sun' equals about 2000 trillion trillion tons!

Problem 1 - Suppose that NGC-290 has a total mass of no more than 500 suns. If it consists of young luminous blue B-type stars with individual masses of 10 suns, and old red super giant M-type stars with individual masses of 30 suns, graph an inequality that shows the number of B and M-type stars in this cluster. Write an inequality that represents this information and solve it graphically.

Problem 2 - Does the combination of 9 B-type stars and 32 M-type stars lead to a possible population solution for this cluster.

Problem 1 - Answer: The equation would be **10 B + 30 M < 500**. To solve it, do the following algebra steps to write this as a linear function in standard 'y = mx+b' form:

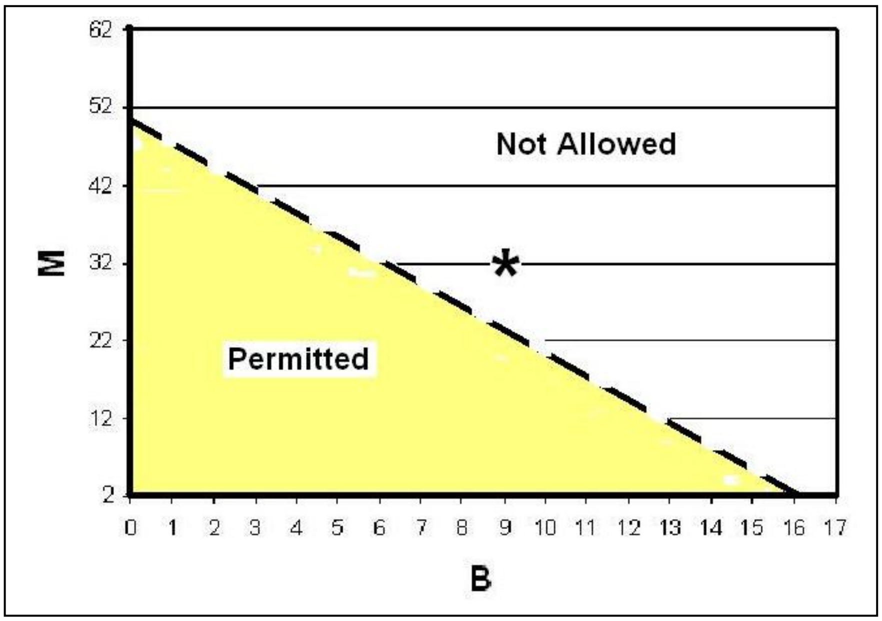
10 B + 30 M < 500
10 B < 500 - 30M or 30 M < 500 - 10 B
B < 50 - 3M or M < 50 - B

Now:
1) Graph the limiting equation B = 50 - 3M (or M = 50 - B)

2) Because it is a '<' rather than a '≤' condition, draw this limiting equation for B as a dotted line to emphasize that the solution does not include points on this line, which would make 10 B + 30 M = 500. Note: you cannot have 'fractions of a star' and you cannot have negative numbers of stars.

3) Shade-in the region below this line which represents all of the possible combinations of B and M that yield a total cluster mass less than 500 suns. The answer for B < 50 - 3M is shown below:

Problem 2 - Answer: No because this combination as a point (9,32) falls outside the permitted (shaded) region of the graph (see graph below with plotted point at (9,32)).



Linear Inequalities in 2 Variables:

Astronomers think that weakly interacting massive particles, called WIMPS, may be a common ingredient to the universe, but so far many different searches have failed to turn up any detections of these hypothetical particles. Do they exist at all?

By combining the data from many different studies, physicists have been able to narrow the possibilities for the masses of these hypothetical particles.

In the problem below, translate the constraint into a 2-variable inequality and graphically solve the combined inequalities to find a possible solution to all of the constraints.

The two variables involved are W, which is the mass of the WIMP particle in multiples of the mass of the proton, and C which is the strength of its interaction with matter.

Constraint 1: $C - 0.098W \leq 2.06$

Constraint 2 : $W > 50$

Constraint 3: $1075 \leq C + 1.075W$

Constraint 4: $C + 0.41W \leq 103$

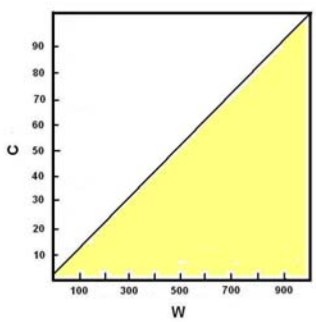
Problem 1 - On a single graph, plot each of these inequalities over the domain W:[1, 1000] and the range C:[1,100] and shade-in the permitted region.

Problem 2 - Which of the following points satisfy all four constraints?

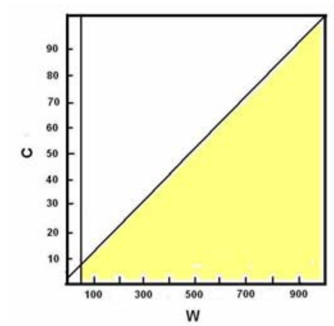
A) (300, 10) B) (200, 15) C) (100, 20) D) (75,5)

Problem 1 - Graph each of these over the domain W:[1, 1000] and the range C:[1,100] and shade-in the permitted region

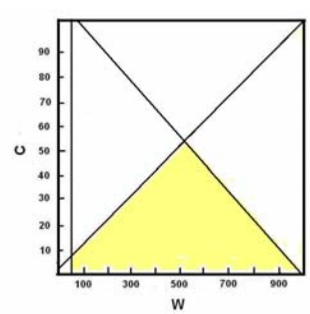
Constraint 1: $C - 0.098W \leq 2.06$



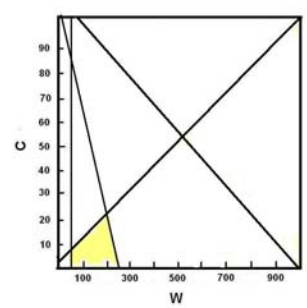
Constraint 2 : $W > 50$



Constraint 3: $1075 \leq C + 1.075W$



Constraint 4: $C + 0.41W \leq 103$



Problem 2 - Which of the following points satisfy all four constraints?

A) (300, 10) B) (200, 15) C) (100, 20) D) (75,5)

Answer: Only **B and D.**

Note: Data is based upon Figure 4 in the paper 'First limits on WIMP dark matter from the XENON10 experiment' by Uwe Oberlack, Journal of Physics Conference series 110 (2008) doi:10.1088/1742-6596/110/6/06/2020

Linear Inequalities: Exoplanet Habitable Zones

Astronomers have discovered over 400 planets orbiting nearby stars, and the search is on for ones that are Earth-like in size. In order for them to also be potential places were living things could exist, these planets must also satisfy several other constraints that have to do with their distance from their star.

Many of the planets detected so far are too close to their star for water to remain in liquid form.

There are four important constraints that determine whether life has a chance on such an Earth-sized planet or not. Two of them define where the Zone of Water, also called the Habitable Zone, can exist. Within this range of planet distances (D) and stellar masses (M), a planet can be warm enough to have liquid water on its surface. Outside this zone, water either freezes (temperature less than 0^{o}C) or boils and turns to steam (temperature greater than 100^{o}C) on the surface of the planet.

Constraint 1: $M - 0.8D \leq 0.12$
Constraint 2: $M - 1.2D \geq 0.18$

The third constraint defines the maximum distance for which the planet's rotation period will be 'locked' with its star as it orbits, so that it always has the same hemisphere facing its star. This is a bad situation because the planet will have the same half of its surface in perpetual night time with very cold, below freezing, temperatures.

Constraint 3: $M - 3.3D \geq -1.3$

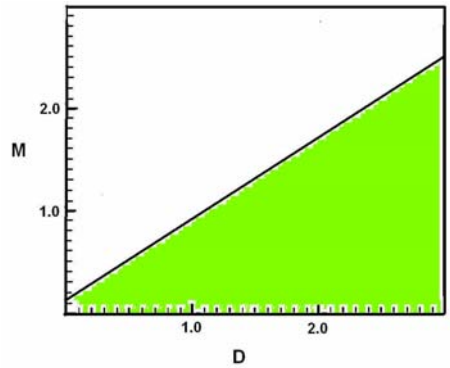
Create a graph for the mass of the star (M) and the distance to the planet (D) with the domain D:[0.0, 3.0] and range M:[0.0, 2.0] in intervals of 0.1. The star masses are in multiples of our sun's mass (2×10^{30} kg) and the planet distances are in multiples of the Earth-Sun distance called the Astronomical Unit ('D=2' means 2 x 150 million kilometers).

Problem 1 - From the three constraints, shade-in the regions for which water is lower than the freezing point and higher than boiling point, and where the planet's rotation is locked in synchrony with its orbital period.

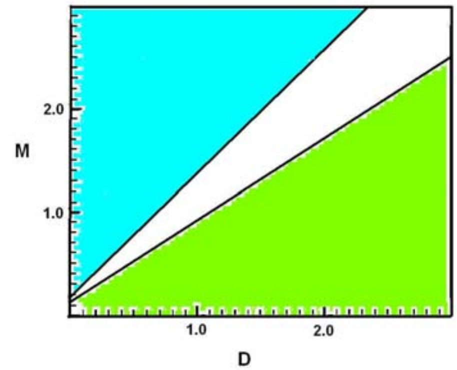
Problem 2 - Which of these hypothetical Earth-sized planets may be in the unshaded 'Habitable Zone' for its star? A) Osiris (D= 2.0 AU, M=1.0); B) Hades (D=0.5AU, M=2.0) C) Oceania (D=2.0 AU, M=2.0)

Problem 1 - From the three constraints, shade-in the region for which water is between the freezing point and boiling point, and where the planets rotation is not locked in synchrony with its orbital period.

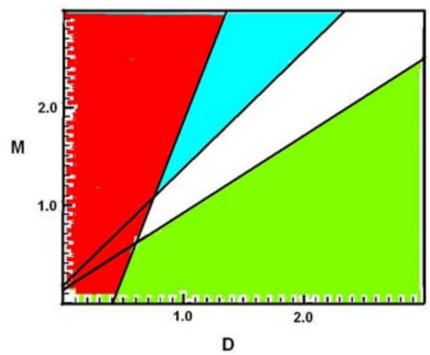
Constraint 1: (Green)



Constraint 2: (Blue).



Constraint 3: (Red)



The un-shaded area (white) in the graph for Constraint 3 now shows the solution space

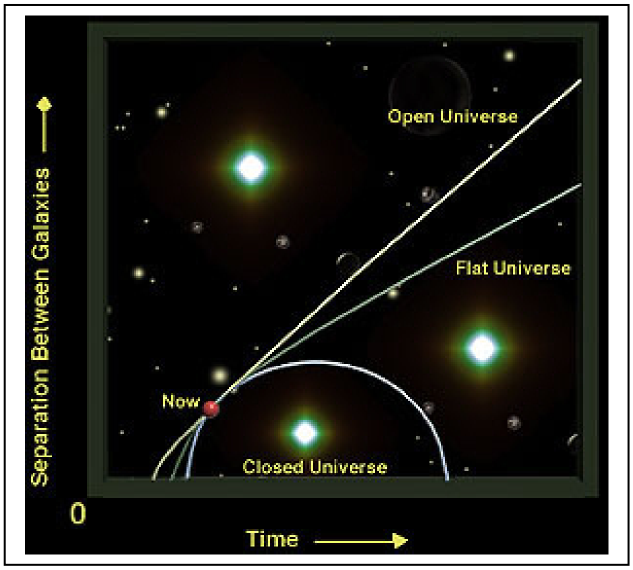
Problem 2 - Which of these hypothetical Earth-sized planets may be in the 'Habitable Zone' for its star? A) Osiris (D= 2.0 AU, M=1.0); B) Hades (D=0.5AU, M=2.0) C) Oceania (D=2.0 AU, M=2.0)

Answer: A) Osiris is located in the zone where water is permanently in ice form and is outside the Habitable Zone for its star.

B) Hades is located in the 'red zone' where water is above its boiling point, and the planet is permanently locked so that the same face of the planet always faces its star.

C) Oceania is located in the Habitable Zone of its star. And is far enough from its star that it rotates normally and is rotation period is not 'locked' in synchrony with its orbital year.

Piecewise Functions



The universe has gone through three different stages of expansion soon after the Big Bang. Astronomers call these stages the Inflationary Era, Radiation Era, the Matter Era.

The size of the universe is determined by the separations between typical objects, and can be represented by mathematical models that are based on the physical equations that govern the behavior of matter, energy and gravity.

The expansion of the universe can be defined by the following piecewise function, where the variable t is measured in seconds from the Big Bang:

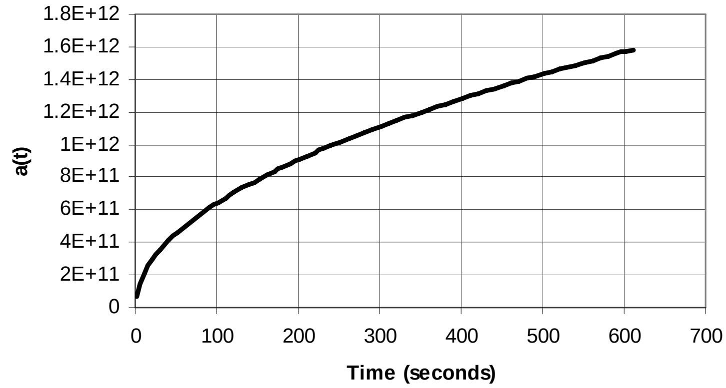
$$a(t) = \begin{cases} 2.2x10^{-29} e^{(10^{35} t)} & 10^{-35} < t < 10^{-33} \quad \text{Inflation Era} \\ 6.4x10^{10} \sqrt{t} & 10^{-33} < t < 9.3x10^{12} \quad \text{Radiation Era} \\ 7700\, t^{\frac{4}{3}} & 9.3\text{x}10^{12} < t < 4.2x10^{17} \quad \text{Matter Era} \end{cases}$$

Problem 1 – What is the graph of a(t) between 1 second and 10 minutes after the Big Bang? (Hint: Convert time interval into seconds)

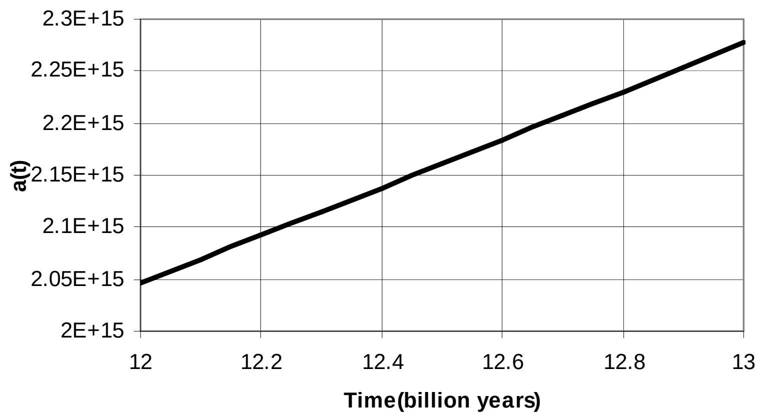
Problem 2 - What is the graph of a(t) between 12 and 13 billion years after the Big Bang? (Hint: Convert time interval into second: 1 year = $3.1\text{x}10^{7}$ seconds)

Problem 3 – By what factor does a(t) change as the time since the Big Bang increases by a factor of 10 during each era?

Problem 1 – What is the graph of a(t) between 1 second and 10 minutes after the Big Bang? (Hint: Convert time interval into seconds)



Problem 2 - What is the graph of a(t) between 12 and 13 billion years after the Big Bang? (Hint: Convert time interval into seconds: 1 year = 3.1×10^7 seconds)



Problem 3 – By what factor does a(t) change as the time since the Big Bang increases by a factor of 10 during each era?

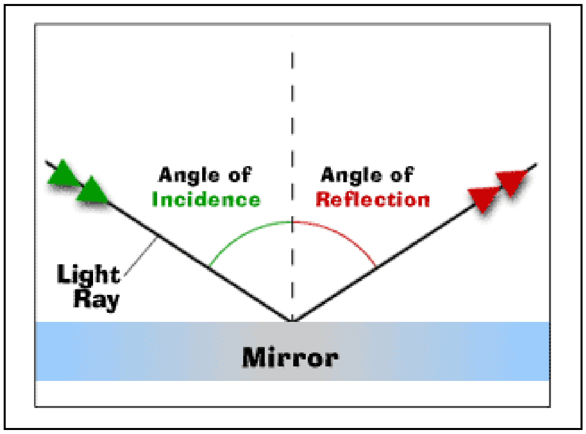
Answer: Inflation: a(t) changes by e^{10} = **22,026 times**

Radiation: a(t) changes by $10^{1/2}$ = **3.2 times**

Matter: a(t) changes by $10^{4/3}$ = **21.5 times**

Absolute Value Functions

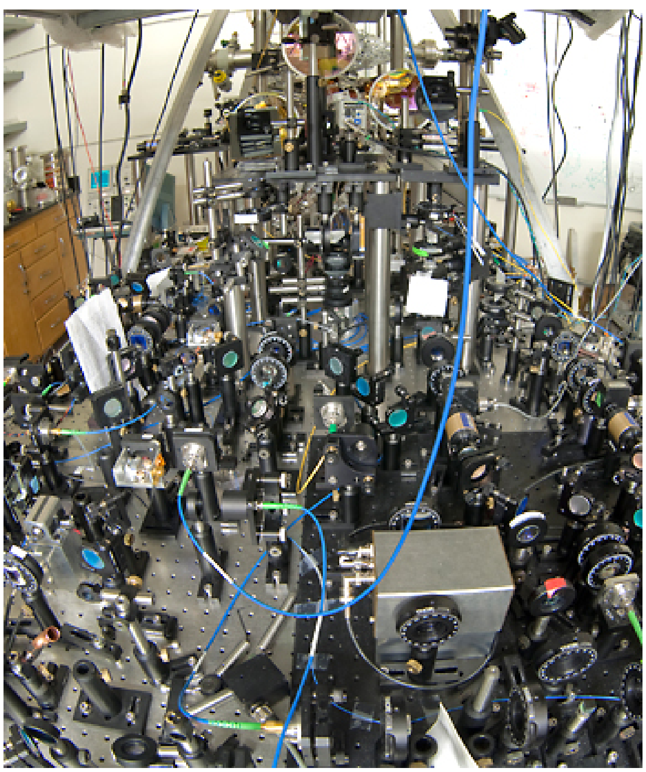
2.8.1



When a light wave reflects from a surface, the distance of the crest from the surface follows an absolute-value function.

As measured from the vertical axis at the point of reflection, the angle that the incident light wave makes to the vertical axis is equal to the angle made by the reflected wave.

Problem 1 – The equation of a light ray is given by $y = |x-5| + 3$. Graph this equation over the range y:[-6,+12]. A) What is the equation for the reflecting surface? B) How far does the light ray get from the vertical axis at a height of 10 centimeters?

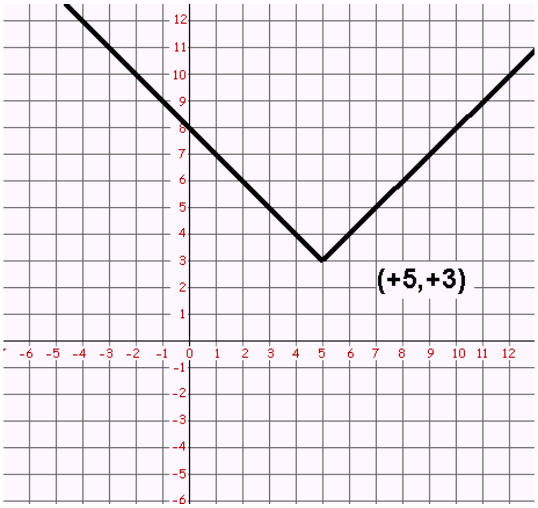


This jumble of hundreds of mirrors and lenses is used to control six beams of laser light being used in an experiment to test Einstein's theory of relativity.

An astronomer is designing an optical interferometer to measure the sizes of nearby stars. The optical bench like the one shown to the left, contains many mirrors and lenses to manipulate the incoming light rays from the stars so that they can be analyzed by the instrument.

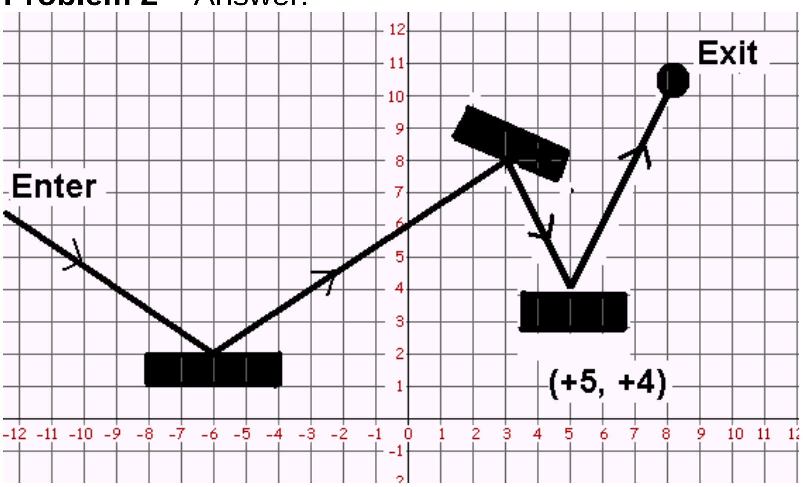
Problem 2 – An incoming light ray follows a path defined by the equation $y = \frac{2}{3}|x+6| + 2$ where all units are in centimeters. Two mirrors are placed at (-6,+2) and (+3, +8). Where does the third mirror have to be placed along the line x = +5 so that the light ray is reflected to a point at (+8,+10)? (Solve graphically or mathematically)

Problem 1 – Answer:



A) The reflecting surface is at y=+3. B) Solve for y=+10 to get x = -2 and +12. The farthest distance, d, from the vertex is d = 7 centimeters.

Problem 2 – Answer:



Answer: Students may solve this graphically as in the figure above. The vertex of the incoming ray is at (-6,+2) which matches the coordinate of the first mirror. The second mirror is at (+3,+8) which is a solution of y = 2/3|3+6| + 2 = +8 so it is located along the line defined by y over the domain x:[-6, +3]. For this light ray to get to the point (+8,+10) from (+3,+8) both of these points must be solutions of the second absolute-value function y = a|x-5| + b whose vertex is defined by the point (+5, b).

Point 1: +10 = a|8-5| + b so 10 = 3a + b
Point 2: + 8 = a|3-5| + b so 8 = 2a + b

Using substitution: b = 8-2a and so 10 = 3a + (8-2a) and a=2 so b = 4
Y = 2|x-5| + 4 so the mirror is placed at (+5, +4) or at **y=+4** along x = +5.