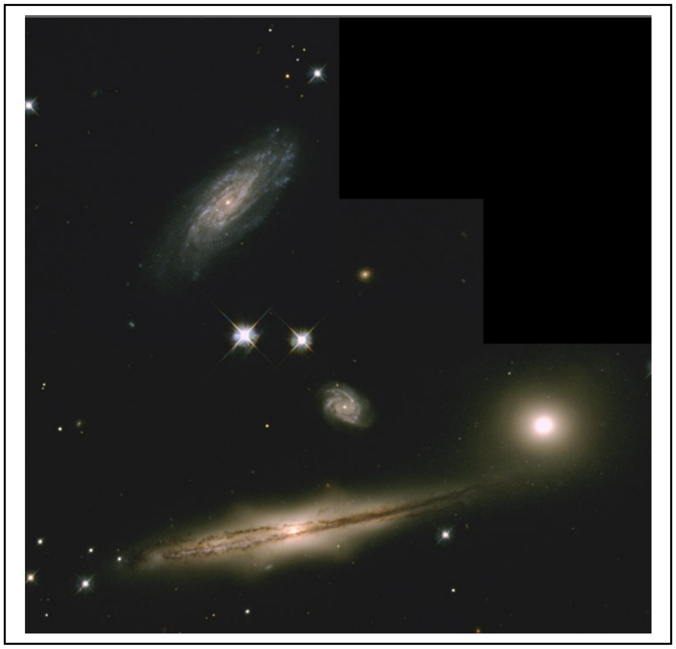
# The Distance Formula in 2 Dimensions 10.1.1



The Hubble Space Telescope image to the left shows the Hickson Compact Group HCG87. It is located 400 million light from Earth in the constellation Capricornus, and consists of five galaxies in the group.

Astronomers use digital photography to 'image' distant stars and galaxies. From this information, they can determine how far apart various objects are. This is an important quantity needed to gauge the physical sizes of the objects and systems they are investigating.

The basic distance formula in 2-dimensions is:

$$D^2 = (x_b - x_a)^2 + (y_b - y_a)^2$$

where the coordinates for the two points A and B are given by $(x_a, y_a)$ and $(x_b, y_b)$, and D is the distance between the two points.

**Problem 1** - On a digital photograph, an astronomer measures the <u>pixel coordinates</u> of two colliding galaxies. Galaxy A is located at (23.4, 105.4) and Galaxy B is located at (35.9, 201.0). To two significant figures, how many pixels apart on the photograph are the galaxies located?

**Problem 2** - Digital photographs of objects in the sky can only capture the way that objects appear to be located in space, not the way that they actually are located in three-dimensions. This is a familiar projection effect. Astronomers measure distances on the 2-dimensional sky plane in terms of angular degrees, minutes of arc and seconds of arc. If the camera used to image these galaxies has a resolution of 0.5 arcseconds per pixel, how far apart are Galaxy A and B in terms of arcseconds?

**Problem 1** – On a digital photograph, an astronomer measures the pixel coordinates of two colliding galaxies. Galaxy A is located at (23.4, 105.4) and Galaxy B is located at (35.9, 201.0). To two significant figures, how many pixels apart on the photograph are the galaxies located?

Answer: $D^2 = (35.9-23.4)^2 + (201.0-105.4)^2$
$D^2 = 156.25 + 9139.36$
$D = (9295.61)^{1/2}$
$D = 72.77$ pixels.

To two significant figures this becomes **73 pixels**.

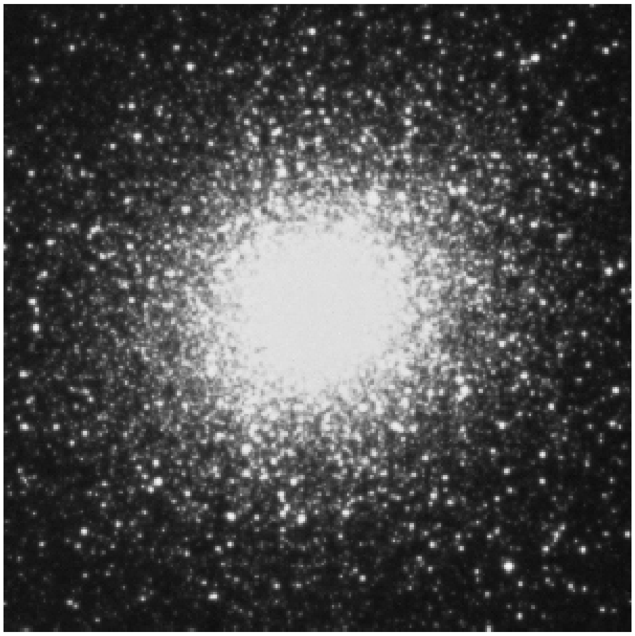
**Problem 2** - If the camera used to image these galaxies has a resolution of 0.5 arcseconds per pixel, how far apart are Galaxy A and B in terms of arcseconds?

Answer: The angular distance is just D = 73 pixels x (0.5 arcseconds/1 pixel) = **36.5 arcseconds**.

Note: At the distance of this cluster, which is 400 million light years, an angular separation of 1 arcsecond corresponds to 400 million x 1/206265 = 1900 light years, so the two galaxies being 36.5 arcseconds apart in the sky, are actually 36.5 x 1900 = 69,300 light years apart, which is less than the diameter of the Milky Way galaxy!

# The Distance Formula in 3 Dimensions





Messier-13 (NASA/Hubble ST)

Astronomers can accurately measure the location of an object in the sky, and from the distance to the object, they can determine the location of the object in 3-dimensional space relative to our Earth.

From this information, they can determine how far apart various objects are. This is an important quantity needed to gauge the physical sizes of the objects and systems they are investigating.

The basic distance formula in 3 -dimensions is:

$$D^2 = (x_b - x_a)^2 + (y_b - y_a)^2 + (z_b - z_a)^2$$

where the coordinates for the two points A and B are given by ($x_a$, $y_a$, $z_a$) and ($x_b$, $y_b$, $z_b$), and D is the distance between the two points. An astronomer wants to know the shortest distance in kiloparsecs between three globular star clusters orbiting the Milky Way:

Globular cluster Messier-13 is located at (+5.0, +4.3, +4.0),
Globular cluster 47 Tucana is located at (-2.9, +2.9, +3.0 )
Globular cluster Messier-15 is located at (+11.3, -5.8, +5.9).

**Problem 1** - Which two clusters are closest to each other?

**Problem 2** - Which cluster is located closest to our sun at (0,0,0)?

Globular cluster Messier-13 is located at (+5.0, +4.3, +4.0),
Globular cluster 47 Tucana is located at (-2.9, +2.9, +3.0 )
Globular cluster Messier-15 is located at (+11.3, -5.8, +5.9).

**Problem 1** –Which two clusters are closest to each other?

Answer: There are 3 clusters that can be paired as follows:

M-13 and 47 Tuc

$D^2 = (5.0+2.9)^2 + (4.3-2.9)^2 +(4.0-3.0)^2$ $D^2 = 65.37$ D = 8.08 kiloparsecs

M-13 and M-15

$D^2 = (5.0-11.3)^2 + (4.3+5.8)^2 +(4.0-5.9)^2$ $D^2 = 145.31$ D = 12.05 kiloparsecs

M-15 and 47 Tuc

$D^2 = (-11.3 +2.9)^2 + (-5.8-2.9)^2 +(5.9-3.0)^2$ $D^2 = 154.66$ D = 12.43 kiloparsecs

So the distance between **Messier-13 and 47 Tucana is the shortest**.

**Problem 2** - Which cluster is located closest to our sun at (0,0,0)?

Answer:

Messier 13:

$D^2 = (5.0-0.0)^2 + (4.3-0.0)^2 +(4.0-0.0)^2$ $D^2 = 59.49$ D = 7.7 kiloparsecs

47 Tucana:

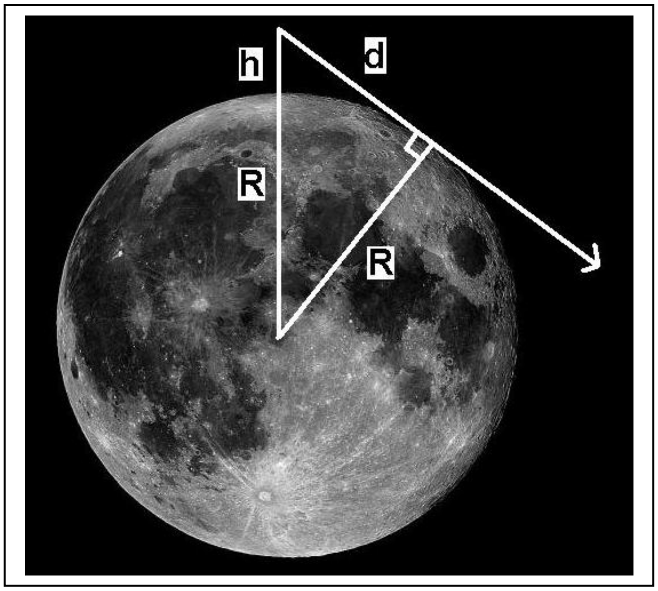
$D^2 = (-2.9-0.0)^2 + (2.9-0.0)^2 +(3.0-0.0)^2$ $D^2 = 25.82$ D = 5.1 kiloparsecs

Messier-15:

$D^2 = (11.3-0.0)^2 + (-5.8-0.0)^2 +(5.9-0.0)^2$ $D^2 = 196.14$ D = 14.0 kiloparsecs

So **47 Tucana is closest**.

# Calculating the Distance to the Horizon




An important quantity in planetary exploration is the distance to the horizon. This will, naturally, depend on the diameter of the planet (or asteroid!) and the height of the observer above the ground.

Another application of this geometry is in determining the height of a transmission antenna on the Moon in order to insure proper reception out to a specified distance.

The figure shows how the horizon distance, d, is related to the radius of the planet, R, and the height of the observer above the surface, h.

**Problem 1:** The triangle shown in the figure is a right triangle with the distance to the tangent point, called the Line-of-Sight (LOS), given by the variable d. What is the total length of the hypotenuse of the triangle, which represents the distance of the Observer from the center of the moon?

**Problem 2** - Use the Pythagorean Theorem to derive the formula for the LOS horizon distance, d, to the horizon tangent point given R and the length of the hypotenuse.

**Problem 3**: For a typical human height of 2 meters, what is the horizon distance on A) Earth (R=6,378 km); B) The Moon (1,738 km)

**Problem 4:** A radio station has an antenna tower 50 meters tall. What is the maximum LOS reception distance in the Moon?

Answer Key:

**Problem 1 – From the figure, the total length is just R + h.**

**Problem 2**: By the Pythagorean Theorem

$d^2 = (R+h)^2 - R^2$

so $d = ( R^2 + 2Rh + h^2 - R^2 )^{1/2}$

and so $\mathbf{d = ( h^2 + 2Rh )^{1/2}}$

**Problem 3:** Use the equation from Problem 1.

A) For Earth, R=6378 km and h=2 meters so

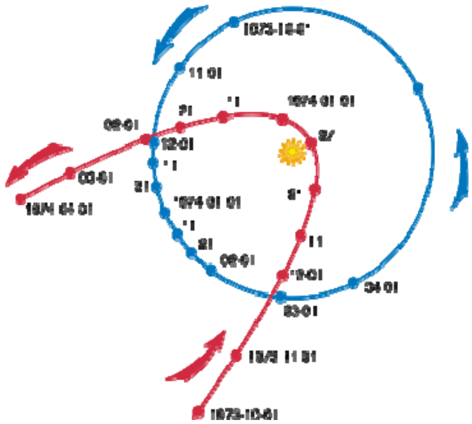
$d = ((2 \text{ meters})^2 + 2 \times 2 \text{ meters} \times 6.378 \times 10^6 \text{ meters}) )^{1/2}$

= 5051 meters or **5.1 kilometers**.

B) For the Moon, R=1,738 km so d = 2.6 kilometers

**Problem 4:** h = 50 meters, R=1,738 km

so d = 13,183 meters or **13.2 kilometers**.



Orbit of Comet Kohoutek in 1973 shown together with circular orbit of Earth. Both orbits have as a common focus at the Sun.

Parabolas are found in many situations in astronomy, from the curve of a telescope mirror to the orbit of a comet.

One of the most important, and useful, features of a parabola is the existence of a 'focus' point that is located in the space interior to the curve, but does not exist on the curve itself. A quadratic equation with vertex at (0, 0) and axis of symmetry along the y-axis, is of the form;

$$y = ax^2$$

The focus of the parabola is located along the axis of symmetry of the parabola, at a point (0,+p) where p is defined by

$$p = 1/(4a)$$

Suppose that the orbit of a comet can be modeled by the formula:

$$y(x) = x^2 – 11x + 24$$

where the coordinate distances x and y are measured in Astronomical Units. (1 A.U. = 150 million kilometers)

**Problem 1** – If the general form for a parabola is $4P(y-k) = (x-h)^2$, based upon the comet's trajectory, what are P, k and h?

**Problem 2** – Graph y(x). Where is the axis of symmetry located?

**Problem 3** – Where is the vertex of the comet orbit located?

**Problem 4** - What are the coordinates of the focus of the comet's orbit?

**Problem 5** – If the Sun is at the focus of the orbit, what is the closest distance that the comet reached from the Sun, called the perihelion distance?

**Problem 1** – If the general form for a parabola is $4P(y-k) = (x-h)^2$, based upon the comet's trajectory, what are P, k and h? Answer: Expanding the general formula we get:

$x^2 - 2xh + h^2 = 4Py - 4Pk$

$y = (x^2 - 2xh + h^2 + 4Pk) / 4P$

$y = (1/4P)\, x^2 - (h/2P)\, x + h^2/4P + k$

Then from $y(x) = x^2 - 11x + 24$ we have term by term:

$1/4P = 1$ so **$P = +0.25$**

$-h/2P = -11$ so $h = 22P$ and **$h = +5.5$**

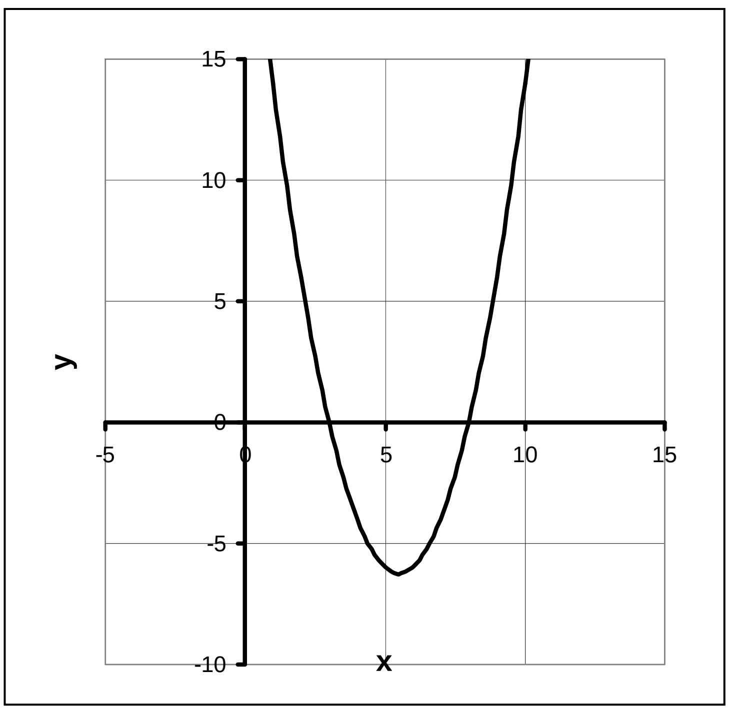
$h^2/4P + k = +24$ so $k = +24 - (5.5)^2$, **$k = -6.25$**

**Problem 2** – Answer: See graph below. Factor y(x) to get $y(x) = (x-8)(x-3)$. The axis of symmetry is along the line <u>half-way</u> between the x-intercepts: **x = +5.5**

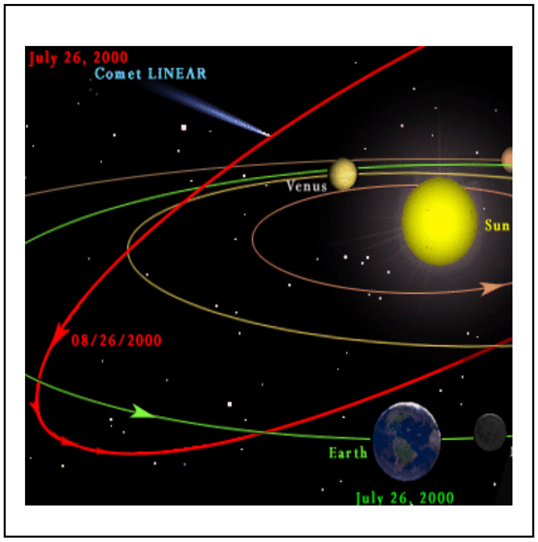
**Problem 3** – Answer: $y(+5.5) = (5.5)^2 - 11(5.5) + 24 = -6\ 1/4$ so the vertex is at **(+5.5, -6.25)**

**Problem 4** – Answer: The focus is along the axis of symmetry at x=5.5, y = -6.25 +p = -6.0 or
**(+5.5, -6.0)**.

**Problem 5** -Answer: This is the same as P which is **1/4 AU**. Note the orbit of the planet Mercury is at 0.35 AU.



# Parabolas




Parabolas are not that common among the orbits of objects in our solar system, but their simple shape can often be used to illustrate many different principles in astronomy and in celestial navigation. Once a comet is spotted, one of the first things astronomers do is to determine the orbit of the comet from the various positions of the comet in the sky.

Within the plane of the orbit of the comet, its parabolic path can be represented by the standard equation for a parabola in 2-dimensions:

$$x^2 = 4\,Py$$

Suppose an astronomer makes the following position measurements of Comet XYZ within it orbital plane:

March 5, 2017 : (+2, +6) June 5, 2017 : (+3, 0) September 5, 2017 : (+4, -4)

The units of the coordinates are in Astronomical Units for which 1 AU = 150 million kilometers; the distance between the Sun and Earth, and a standard distance unit for measuring distances in our solar system.

**Problem 1** – Graph these points on the domain x:[0, +10] and range y:[-10, +10].

**Problem 2** – From the general equation for a parabola, $y = ax^2 + bx + c$, fit these three comet points to a single parabolic orbit by solving for the constants a, b and c.

**Problem 3** – What is the distance between the focus (location of Sun) and the vertex (perihelion of comet)?

**Problem 1** – Graph these points on the domain x:[0, +10] and range y:[-10, +10].
Answer: See graph below.

**Problem 2** – From the general equation for a parabola, $y(x) = ax^2 + bx + c$, fit these three comet points to a single parabolic orbit by solving for the constants a, b and c.
Point 1 : (+2, +6) Point 2 : (+3, 0) Point 3 : (+4, -4)
Answer: Each point must be a solution to y(x), so for three points we have three equations

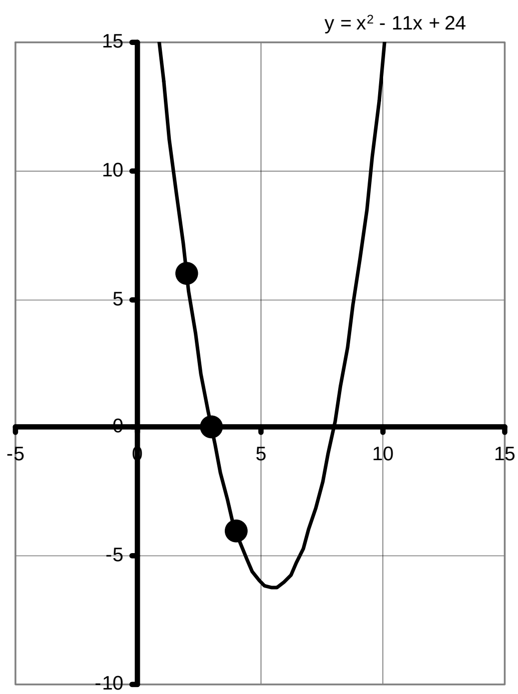
Point 1: $+6 = a(2)^2 + b(2) + c$ so $6 = 4a + 2b + c$
Point 2: $0 = a(3)^2 + b(3) + c$ so $0 = 9a + 3b + c$
Point 3: $-4 = a(4)^2 + b(4) + c$ so $-4 = 16a + 4b + c$

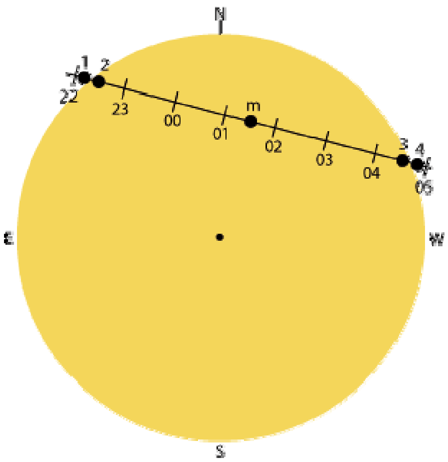
Solve this system of equations for a, b and c to get

a = 1.0 b= -11 c= +24 and so **$y(x) = x^2 - 11x + 24$**

**Problem 3** – What is the distance between the focus (location of Sun) and the vertex (perihelion of comet)?

Answer: Since for a parabola $4py = x^2$, and from y(x) we have a = 1, then p = ¼. The perihelion distance is **¼ AU or 0.25 AU.**






During the Transit of Venus on June 5, 2012, the planet Venus will move across the face of the Sun. This will be the last time this phenomenon will be visible from Earth until December 10, 2117!

Depending on your location, the dark disk of Venus will travel in a straight line across the disk of the Sun, taking different amounts of time. The figure to the left shows one such possibility lasting about 7 hours.

The standard equation of a circle centered at (h,k) is given by:

$$r^2 = (x - h)^2 + (y-k)^2$$

**Problem 1** – The diameter of the Sun at that time will be 1890 seconds of arc. Write the equation for the circular edge of the Sun with this diameter if the origin is at the center of the sun's disk.

**Problem 2** – Suppose that from some vantage point on Earth, the start of the transit occurs on the eastern side of the Sun at the point (-667, +669), and follows a path defined by the equation $y = -0.1326x +580.5$. What will be the coordinates of the point it will reach on the opposite, western edge of the Sun at the end of the transit event?

**Problem 3** – If Venus moves across the Sun at a speed of 200 arcseconds/hour, how long with the transit take based on the endpoints calculated in Problem 2?

**Problem 1** – The diameter of the Sun at that time will be 1890 seconds of arc. Write the equation for the circular edge of the Sun with this diameter.

Answer: The radius is 1890/2 = 945, so the formula for a center at (0,0) is
$(945)^2 = x^2 + y^2$ **so** $\mathbf{x^2 + y^2 = 893{,}025}$

**Problem 2** – Suppose that from some vantage point on Earth, the start of the transit occurs on the eastern side of the Sun at the point (-667, +669), and follows a path defined by the equation $y = -0.1326x + 580.5$. What will be the coordinates of the point it will reach on the opposite, western edge of the Sun at the end of the transit event?

Answer: We need to find the point where <u>both</u> equations are satisfied at the same time.

Equation 1) $x^2 + y^2 = 893{,}025$ Equation 2) $y = -0.1326x + 580.5$

By substituting Equation 2 into Equation 1 we eliminate the variable 'y' and get

$x^2 + (-0.1326x + 580.5)^2 = 893{,}025$ which simplifies to:
$1.0176\ X^2 - 153.95\ X - 556{,}045 = 0$

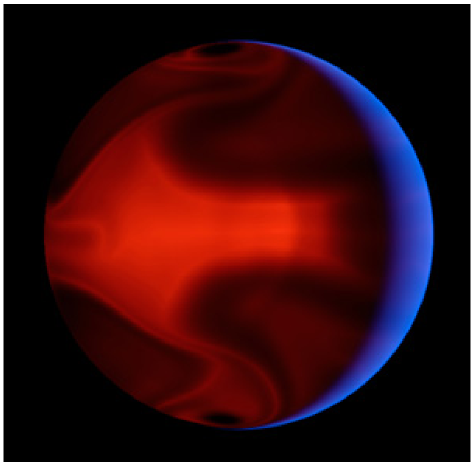
Using the quadratic formula to find the two roots, we get x = 76 +/- 743 and the roots x1 = +819 and x2 = -667. We already know the starting point at x=-667, so the desired point must be x1 = +819 and from the equation for the transit line
Y2 = -0.1326(819)+580.5 = +472.0. **so the two endpoints for the transit are (-667, +669) and (+819, +472).**

**Problem 3** – If Venus moves across the Sun at a speed of 200 arcseconds/hour, how long with the transit take based on the endpoints calculated in Problem 2?

Answer: The distance between the transit endpoints can be found from the Pythagorean Theorem:

$D = ( (819 + 667)^2 + (472 - 669)^2)^{1/2} =$ 1,499 arcseconds.

At a speed of 200 arcseconds/hour, the transit will take 1,499/200 **= 7.5 hours.**



A model of the planet based on NASA's Spitzer Space Telescope measurements.

The exoplanet HD80606b orbits the star HD80606 located 190 light years from the Sun in the constellation Ursa Major. It was discovered in 2001. With a mass of about 4 times that of Jupiter, though slightly smaller in diameter, its elliptical orbit is one of the most extreme discovered so far.

During its 111-day orbit, the planet passes so close to its the planet heats up 555 °C (1,000 °F) in just a matter of hours. This triggers "shock wave storms" with winds that move faster than the speed of sound.

The general equation for an ellipse with its major axis along the x-axis and centered on the point (h,k) is given by:

$$r^2 = \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2}$$

**Problem 1** – The planet has a semimajor axis of 0.45 AU and an eccentricity of 0.933. What is the equation for the planet's orbit if the center of the ellipse is defined to be at the center of the coordinate system (0,0)?

**Problem 2** – With the star at one focus, what is the closest distance the planet comes to its star in millions of kilometers, where 1 AU equals 150 million km? This distance is called the periastron.

**Problem 3** - With the star at one focus, what is the farthest distance the planet comes to its star in millions of kilometers, where 1 AU equals 150 million km? This distance is called the apoastron.

**Problem 4** – Graph the ellipse representing the planet's orbit to scale with the circular orbit of Mercury (R=0.35 AU), Venus (R=0.69 AU) and Earth (R=1.0 AU).

**Problem 1** – The planet has a semimajor axis of 0.45 AU and an eccentricity of 0.933. What is the equation for the planet's orbit?

Answer: Eccentricity is defined as e = c / a, and since a = 0.45 we have c = 0.420
The semi-minor axis is then $b = (a^2 - c^2)^{1/2}$ and can solve this to find the semiminor axis b as $b = (0.45^2 - 0.42^2)^{1/2}$ then b = 0.16
The equation for the planet's orbit for (h,k) = (0,0) is then

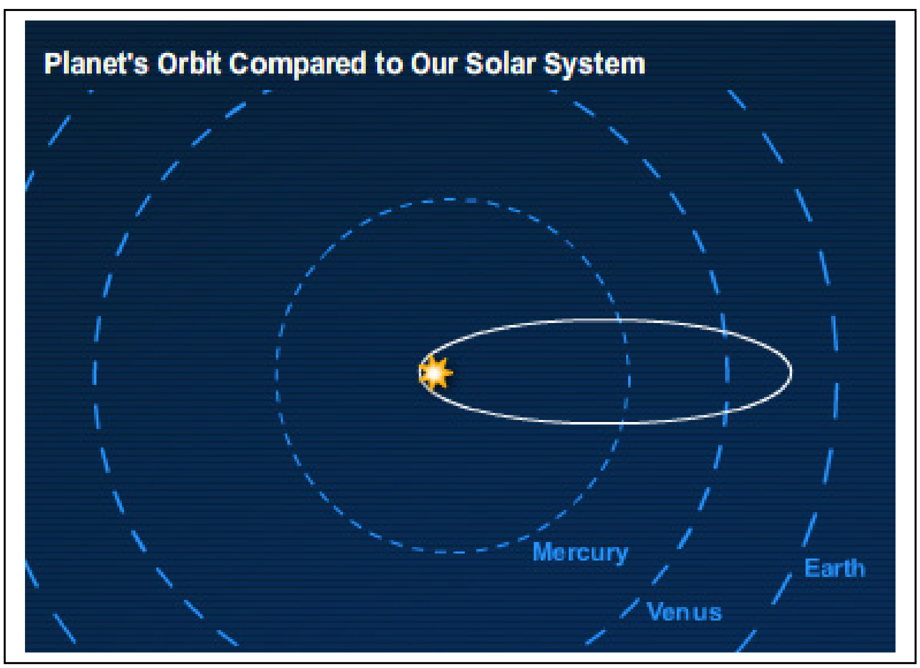
$$\mathbf{x^2/(0.45)^2 + y^2/(0.16)^2 = 1} \quad \textbf{or}$$

$$\mathbf{4.9x^2 + 39y^2 = 1}.$$

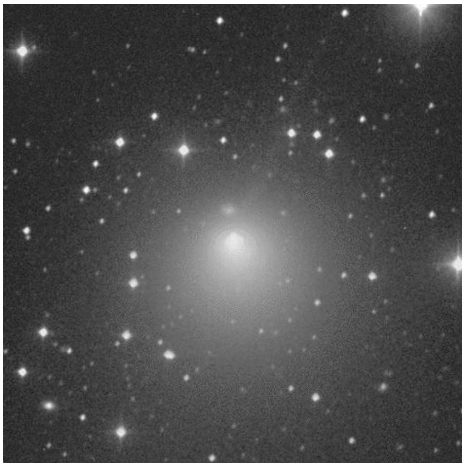
**Problem 2** – With the star at one focus, what is the closest distance the planet comes to its star in millions of kilometers, where 1 AU equals 150 million km? This distance is called the periastron. Answer: Closest distance is given by a – c = **0.03 AU or 4.5 million kilometers.**

**Problem 3** - With the star at one focus, what is the farthest distance the planet comes to its star in millions of kilometers, where 1 AU equals 150 million km? This distance is called the apoastron. Answer: The farthest distance is given by a + c **= 0.87 AU or 130 million km.**

**Problem 4** – Graph the ellipse representing the planet's orbit to scale with the circular orbit of Mercury (R=0.35 AU), Venus (R=0.69 AU) and Earth (R=1.0 AU). Answer; See sketch below for approximate scales.



# Ellipses



This is an image of short-period comet Encke obtained by Jim Scotti on 1994 January 5 while using the 0.91-meter Spacewatch Telescope on Kitt Peak.

Comet Encke was the first periodic comet discovered after Halley's Comet.

A number of sightings since 1786 were put together by Johann Franz in 1819 and revealed that these separate comets were really just one comet!

Although a dramatic sight to see every 3.3 years, the actual nucleus is only 5 kilometers across and consists of an icy 'rock'.

**Problem 1** – The semimajor axis of this comet is a = 2.2 AU, and the orbit has an eccentricity of e = 0.847. What is the semiminor axis and equation for the elliptical orbit?

**Problem 2** – In kilometers, what is the closest (perihelion) and farthest (aphelion) distance from the Sun that the comet reaches in its orbit? (1 AU = 150 million km)

**Problem 1** – The semimajor axis of this comet is a = 2.2 AU, and the orbit has an eccentricity of e = 0.847. What is the semiminor axis and equation for the elliptical orbit?

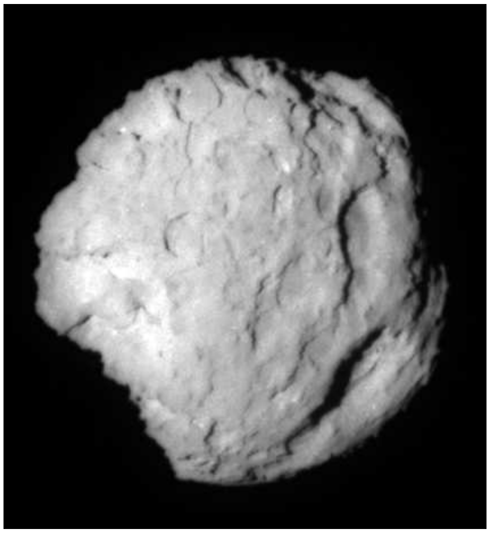
Answer: Since $e = c/a$ we have
$c = 1.86$ so
$b = (a^2 - c^2)^{1/2} = 1.17$ AU.

Formula: $x^2/(2.2)^2 + y^2/(1.17)^2 = 1$

$$0.2x^2 + 0.73\,y^2 = 1$$

**Problem 2** – In kilometers, what is the closest (perihelion) and farthest (aphelion) distance from the sun that the comet reaches in its orbit?

Answer: Perihelion = a – c = 2.2 – 1.17 = 1.03 AU **or 15 million km**.
Aphelion = a + c = 2.2 + 1.17 = 3.37 AU **or 505 million km**.



On January 2, 2004 NASA's Stardust spacecraft flew through the tail of Comet Wild-2 and captured samples of its gas and dust for return to Earth.

The comet has a nuclear body about 5 km across shown in the Stardust image to the left. The many craters detected on its surface come from outgassing events that produce the dramatic tail of the comet. With a total mass of 23 million tons, the mostly water-ice nucleus will be around for a very long time!

An approximate equation for the orbit of this comet is given by the formula: $8.64x^2 + 11.9y^2 = 102.88$. The units for x and y are given in terms of Astronomical Units where 1 AU = 150 million kilometers, which is the average orbit distance of Earth from the Sun.

**Problem 1** - What is the equation of the orbit written in Standard Form for an ellipse?

**Problem 2** – What is the semimajor axis length in AU?

**Problem 3** – What is the semiminor axis length in AU?

**Problem 4** – What is the distance between the focus of the ellipse and the center of the ellipse, defined by c?

**Problem 5** - What is the eccentricity, e, of the orbit?

**Problem 6** – What are the comet's aphelion and perihelion distances?

**Problem 7** – Kepler's Third Law states that the period, P, of a body in its orbit is given by $P = a^{3/2}$ where a is the semimajor axis distance in AU, and the period is given in years. What is the orbital period of Comet Wild-2?

**Problem 1** - What is the equation of the orbit written in Standard Form for an ellipse?
Answer:
$8.64x^2 + 11.9y^2 = 102.88$ Divide both sides by 102.88 to get
$x^2/11.9 + y^2/8.64 = 1$

**Problem 2** – What is the semimajor axis length in AU?
Answer: For an ellipse written in standard form:
$x^2/a^2 + y^2/b^2 = 1$
Comparing with the equation from Problem 1 we get that the longest axis of the ellipse is along the x axis so the semimajor axis is $a^2 = 11.9$ so **a = 3.45 AU**

**Problem 3** – What is the semiminor axis length in AU?
Answer: The semiminor axis is along the y axis so $b^2 = 8.64$ and **b = 2.94 AU**

**Problem 4** – What is the distance between the focus of the ellipse and the center of the ellipse, defined by c?
Answer: $c = (a^2 – b^2)^{1/2}$. With a= 3.45 and b=2.94 we have **c = 1.80**.

**Problem 5** - What is the eccentricity, e, of the orbit?
Answer: The eccentricity e = c/a where so e = 1.8/3.45 and so **e = 0.5**2

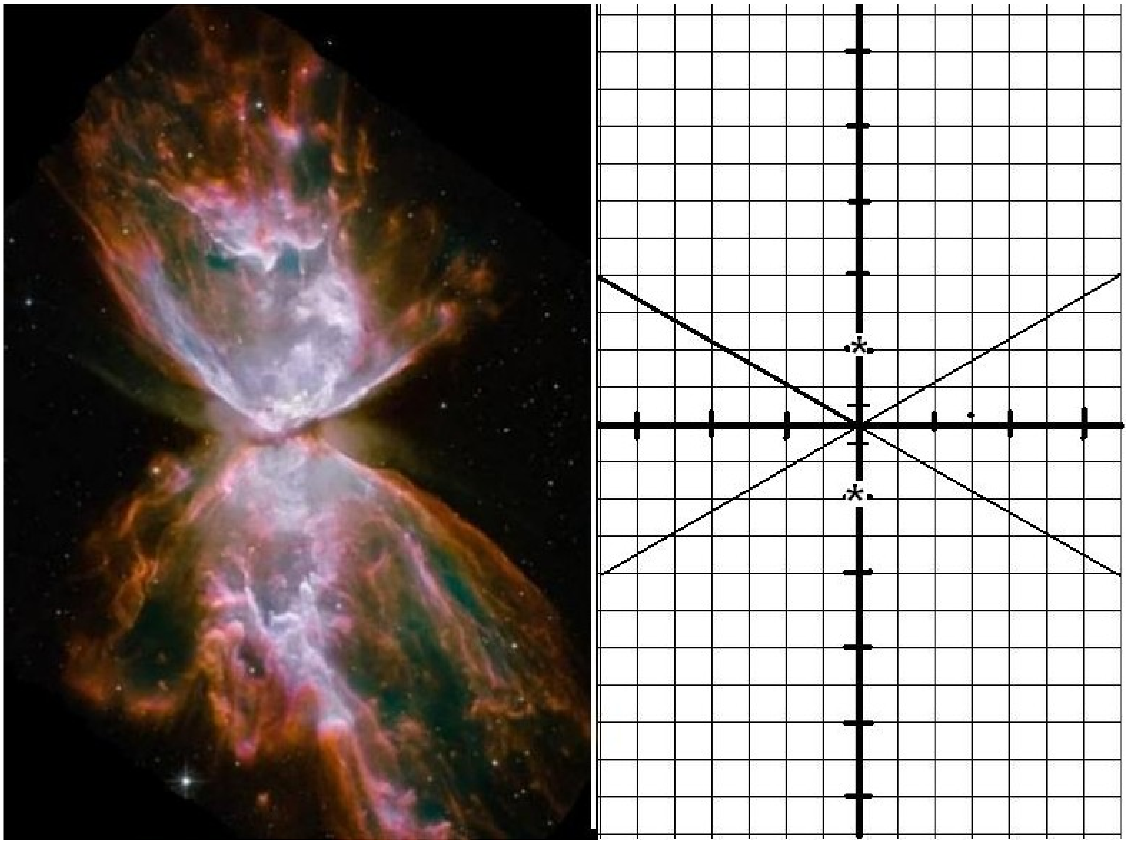
**Problem 6** – What are the comet's aphelion and perihelion distances?
Answer: The closest distance to the focus along the orbit is given by a – c so the perihelion distance is 3.45 – 1.80 = **1.65 AU**. The farthest distance is a + c = 3.45 + 1.86 = **5.25 AU**.

**Problem 7** – Kepler's Third Law states that the period, P, of a body in its orbit is given by $P = a^{3/2}$ where a is the semimajor axis distance in AU, and the period is given in years. What is the orbital period of Comet Wild-2?

Answer: Since a = 3.45 we have $P = 3.45^{3/2}$ = **6.4 years**.

# Hyperbolas

**10.5.1**



The Butterfly Nebula (NGC 6302) is located 3,800 light years from Earth in the constellation Scorpius. It is the remains of an old star that has ejected its outer atmosphere, not as a steady stream of gas, but in bursts of activity. The last of these bursts occurred about 1,900 years ago. The gas travels at over 600,000 miles per hour. The image, obtained from the Hubble Space Telescope is about two light years long.

**Constructing a mathematical model of the shape of NGC-6302.**

**Problem 1** – The pair of lines in the graph above show the approximate locations of the asymptotes of the two 'branches' of the nebular gas shells. The grid has intervals of 0.2 light years. If the foci of the nebular cones are 0.4 light years apart and indicated by the star symbols, what is the equation of the hyperbolic shape of this nebula in standard form centered on the origin of the grid point (0,0) with all units in light years,

$$1 = \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2}$$

**Problem 2 -** What is the equation in the form $cy^2 - dx^2 = 1$ where c and d are constants?

**Problem 1** - The standard form for a vertically-oriented hyperbola with the major axis along the 'y' axis is

$$1 = \frac{y^2}{a^2} - \frac{x^2}{b^2}$$

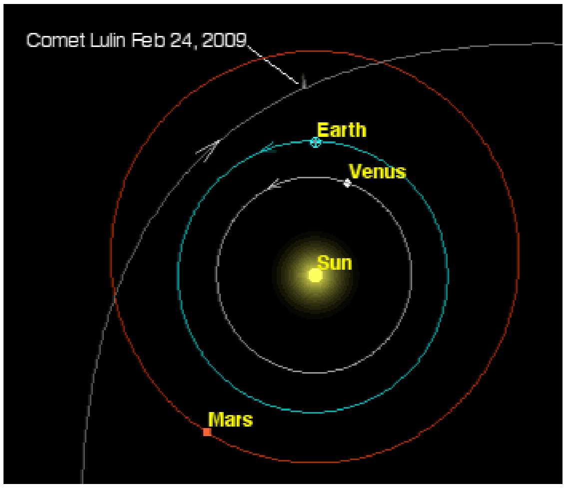
The foci are at (0, +a) and (0,-a) so for the nebula model a = 0.2 light years.

The asymptotes are defined by y = +/- b/a.
From the graph, the slope of the drawn asymptotes between (0,0) and (0.7,0.4) is m = 0.4/0.7 = 0.57 so b/a = 0.57 and since a = 0.2 we have b = 0.57 x 0.2 = 0.11 light years.

The formula for the model in Standard Form is then

$$\frac{y^2}{0.04} - \frac{x^2}{0.012} = 1$$

**Problem 2** - $\mathbf{25y^2 - 83x^2 = 1}$




Comets in hyperbolic orbits around the sun are rarely seen. These objects have speeds that take them far beyond the orbit of Pluto and deep into the distant Oort Comet Cloud; perhaps even interstellar space!

On July 11, 2007, astronomers at the Lulin Observatory in Taiwan discovered the first such comet in recent times. The approximate formula for this comet is

$$4x^2 - y^2 = 4$$

**Problem 1** – What is the formula for the comet in Standard Form?

**Problem 2** – What are the equations for the asymptotes?

**Problem 3** – What is the perihelion of the comet defined as the distance between the vertex and the focus?

**Problem 4** – To two significant figures, which of these points are located on the orbit of the comet: (+1.5, +2.2), (+2.2, +3.9), (+2.7, +6.0), (+3.3, +6.3)?

**Problem 5** – How far is the point (+2.2,+3.9) from the Sun, which is located at the focus of the hyperbola in the domain $x > 0$?

**Problem 1** – What is the formula for the comet in Standard Form?
Answer:

$$\frac{x^2}{1^2}-\frac{y^2}{2^2}=1$$

**Problem 2** – What are the equations for the asymptotes?
Answer:

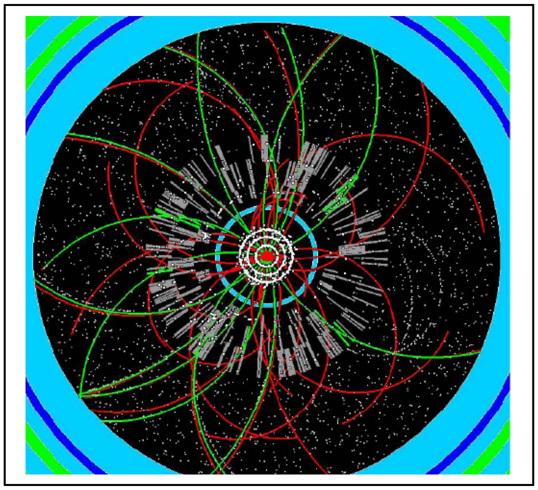
y = +/- b/a x so **y = +2x y = -2x**

**Problem 3** – What is the perihelion of the comet defined as the distance between the vertex and the focus?
Answer: p = c – a and c = $(a^2 + b^2)^{1/2}$ = 2.2 so p = 2.2 – 1.0 = **1.2**

**Problem 4** – To two significant figures, which of these points are located on the orbit of the comet: (+1.5, +2.2), (+2.2, +3.9), (+2.7, +6.0), (+3.3, +6.3)?

Answer: (+1.5, +2.2) : $4(1.5)^2 - (2.2)^2$ = 9.0 – 4.8 = 4.2 = 4 **yes**
(+2.2, +3.9) : $4(2.2)^2 - (3.9)^2$ = 19.4 – 15.2 = 4.2 = 4 **yes**
(+2.7, +6.0) : $4(2.7)^2 - (6.0)^2$ = 29.2 – 36.0 = -6.8 = -7 **no**
(+3.3, +6.3) : $4(3.3)^2 - (6.3)^2$ = 43.6 – 39.7 = 3.9 = 4 **yes**

**Problem 5** – How far is the point (+2.2,+3.9) from the Sun, which is located at the focus of the hyperbola?
Answer: The focus is at (+1,0). $d^2 = (2.2-1.0)^2 + (3.9)^2$ so **d = 4.08**



When the Large Hadron Collider begins its search for exotic particles, it will generate trillions of 'images' showing the tracks of particles through its enormous detectors. The figure to the left shows the tracks passing through the cavernous ATLAS detector whose circular cross section measures 24-meters in diameter.

Particle tracks are mapped out through the detector's volume by individual sensors only a few cubic centimeters in volume. Enormous computing power is needed to keep up with the trillions of tracks generated every second.

Suppose that a collision takes place at the center of the ATLAS detector (0,0), and detectors measure the location, speed and direction of the resulting 'shower' of tracks. In one instance, a mystery particle was created that traveled from the origin to a point (+3 meters,+3 meters). At that location, the mystery particle then decayed into two new particles, A and B, that traveled from this point to the outer circumference before exiting the ATLAS detector.

**Problem 1** – If the track of particle A was determined to be $y = -0.847x + 5.54$, and the track of particle B was determined to be $y = 0.51x + 1.48$, with x and y measured in meters, at what coordinates (x,y) will these particles show up on the circumference of the outer ring assuming they continue to travel along straight lines in the upper half plane of the detector?

**Problem 2** – How far did particles A and B travel in getting from their origin point at (+3, +3) to the outer ring?

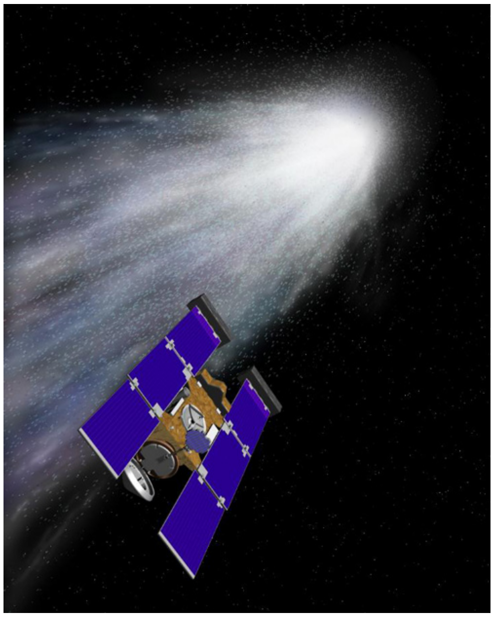
**Problem 3** – If particle A was traveling at a speed of 290,000 km/sec, and particle B was traveling at 230,000 km/sec, how long did it take them to reach the outer ring of ATLAS?

**Problem 1** – Answer: The equation for the outer circular ring is $x^2 + y^2 = 144$
First determine where particle A will appear by elimination of the variable y:
$x^2 + (-0.847x+5.54)^2 = 144$ simplifying to get $1.717x^2 – 9.385x – 113.31 = 0$
Then solve this using the quadratic formula to get the two roots:
$x = 9.385/3.434 +/- (1/3.434) (88.078 -4(1.717)(-113.31)^{1/2}$
$x = 2.73 +/- 8.57$
So x1 = +11.3 x2 = -5.8 then from the formula for the path of particle A:
$y = -0.847x + 5.54$ we have y1 = -4.0 and y2 = +10.5
There are two possible points where particle A could have appeared on the circumference (+11.3, -4.0) and (-5.8, +10.5). **Only (-5.8, +10.5) is located in the upper half-plane**.

Next, do the same analysis to find the points for particle B:
$x^2 + (0.51x+1.48)^2 = 144$ simplifying to get $1.26x^2 +1.5x – 141.8 = 0$
Then solve this using the quadratic formula to get the two roots:
$x = -1.5/2.52 +/- (1/2.52) (2.25 -4(1.26)(-141.8)^{1/2}$
$x = - 0.59 +/- 10.63$
So x1 = +10.0 x2 = -11.2 then from the formula for the path of particle B:
$y = 0.51x + 1.48$ we have y1 = +6.6 and y2 = -4.2
There are two possible points where particle B could have appeared (+10.0, +6.6) and (-11.2, -4.2). **Only (+10.0, +6.6) is located in the upper half-plane.**

**Problem 2** – How far did particles A and B travel in getting from their origin point at (+3, +3) to the outer ring?
Answer: For particle A, use the distance formula between Point1: (+3, +3) and Point2: (+5.8, +10.5) to get $d = (7.8 + 56.3)^{1/2}$ = **8.0 meters**.
For particle B, use the distance formula between Point1: (+3, +3) and Point2: (+10.0, +6.6) to get $d = (49 + 13)^{1/2}$ = **7.9 meters**.

**Problem 3** – If particle A was traveling at a speed of 290,000 km/sec, and particle B was traveling at 230,000 km/sec, how long did it take them to reach the outer ring of ATLAS?
Answer: Since Time = Distance/Speed, and S = 290,000 km/s , D = 8.0 meters or 0.008 km, particle A took T = 0.008 /290,000 = **$2.8 \times 10^{-8}$ sec or 28 nanoseconds**.
Particle B took T = 0.0079/230000 = **$3.4 \times 10^{-8}$ seconds or 34 nanoseconds**.



On January 4, 2004 NASA's Stardust spacecraft passed by Comet Wild-2 and captured samples of its cometary tail.

This comet has been studied for many years by astronomers who know its detailed orbit shape and location. This allows the space craft to be launched years before the encounter and reach its destination to within a few kilometers or less of where the comet is located.

Determining the orbits of comets and asteroids is an important tool in predicting where they will be in the future.

**Problem 1** – Comet Wild-2 was observed by astronomers at the same location in the sky exactly 6.54 years apart. Kepler's Third Law states that $P^2 = a^3$ where **P** is the objects period in years and **a** is the semimajor axis of the orbit in Astronomical Units. If 1 'AU' is the distance of Earth from the Sun (150 million km), what is the semimajor axis, a, of the elliptical orbit of Comet Wild-2?

**Problem 2** – The astronomers have, over the years, collected together many measurements of the comet's position along its orbit. If the coordinates of one of these positions is (+1.9,+2.5) in units of AU
s, what is the semiminor axis of the orbit?

**Problem 3** – What will be the perihelion of the comet defined as d = a – c where c = $(a^2 - b^2)^{1/2}$ ?

**Problem 1** – Comet Wild-2 was observed by astronomers at the same location in the sky exactly 6.54 years apart. Kepler's Third Law states that $P^2 = a^3$ where **P** is the objects period in years and **a** is the semimajor axis of the orbit in Astronomical Units. If 1 'AU' is the distance of Earth from the Sun (150 million km), what is the semimajor axis of the elliptical orbit of Comet Wild-2?

Answer: $a = (6.54)^{2/3}$ so a = **3.5 AU**.

**Problem 2** – The astronomers have, over the years, collected together many measurements of the comet's position along its orbit. If the coordinates of one of these positions is (+1.9,+2.5). What is the semiminor axis of the orbit?

Answer: From the Standard Formula for an ellipse and the semiminor axis distance a = 3.5 we have

$x^2/(3.5)^2 + y^2/b^2 = 1$ then for x=1.9 and y=2.5 we have
$(1.9)^2/(3.5)^2 + (2.5)^2/b^2 = 1$ so $0.29 + 6.25/ b^2 = 1$

And so
$b= (6.25/(1 - 0.29))^{1/2}$
= **2.5 AU**

**Problem 3** – What will be the perihelion of the comet defined as d = a – c where $c = (a^2 - b^2)^{1/2}$ ?

Answer: $c = (3.5^2 - 2.5^2)^{1/2} = 2.4$ AU. Then the perihelion distance
d = 3.5 – 2.4
= **1.1 AU**.

This historic image of the nucleus of Halley's Comet by the spacecraft Giotto in 1986 reveals the gases leaving the icy body to form the tail of the comet.

Once astronomers discover a new comet, a series of measurements of its location allows them to calculate the orbit of the comet and predict when it will be closest to the Sun and Earth.

Although the orbit is actually described by a 3-dimensional equation, the orbit exists within a 2-dimensional plane so the actual shape can be reduced to an elliptical orbit in only 2-dimensions.

**Problem 1** – To improve the accuracy of their elliptical orbit 'fit', astronomers measured three positions of Halley's Comet along its orbit. The x and y locations in its orbital plane are given in units of the Astronomical Unit, which equals 150 million km. The two positions are (+10, +4), (+14, +3) and (+16,+2). What are the three equations for the elliptical orbit based on these three points, written as quadratic equations in a and b, which are the lengths of the semimajor and semiminor axis of the ellipse?

**Problem 2** – Solve the system of three quadratic equations for the ellipse parameters a and b.

**Problem 3** – What is the orbit period of Halley's Comet from Kepler's Third Law is $P^2 = a^3$ where a is in Astronomical Units and P is in years?

**Problem 4** – The perihelion of the comet is defined as d = a – c where c is the distance between the focus of the ellipse and its center. How close does Halley's Comet come to the sun in this orbit in kilometers?

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Answer: The standard formula for an ellipse is $x^2/a^2 + y^2/b^2 = 1$ so we can re-write this as $b^2x^2 + a^2y^2 = a^2b^2$.

Then for Point 1 we have
$10^2b^2 + 4^2a^2 = (ab)^2$ so $\mathbf{100b^2 + 16a^2 = (ab)^2}$. Similarly for Point 2 and Point 3 we have
$14^2b^2 + 3^2a^2 = (ab)^2$ so $\mathbf{196b^2 + 9a^2 = (ab)^2}$ and
$16^2b^2 + 2^2a^2 = (ab)^2$ so $\mathbf{256b^2 + 4a^2 = (ab)^2}$

**Problem 2** – Solve the system of three quadratic equations for the ellipse parameters a and b.
Answer:
$100b^2 + 16a^2 = (ab)^2$
$196b^2 + 9a^2 = (ab)^2$
$256b^2 + 4a^2 = (ab)^2$
Difference the first pair to get $7a^2 = 96b^2$ so $a^2 = (96/7)b^2$.
Substitute this into the first equation to eliminate $b^2$ to get
$(700/96) + 16 = (7/96)a^2$ or $a^2 = 2236/7$ and so **a = 17.8 AU**.
Then substitute this value for a into the first equation to get
$5069 = 217\, b^2$ and so **b = 4.8 AU**.

**Problem 3** – What is the orbit period of Halley's Comet from Kepler's Third Law is $P^2 = a^3$ where a is in Astronomical Units and P is in years? Answer: $P = a^{3/2}$ so for a = 17.8 AU we have **P = 75.1 years**.

**Problem 4** – The perihelion of the comet is defined as d = a – c where c is the distance between the focus of the ellipse and its center. How close does Halley's Comet come to the sun in this orbit in kilometers?
Answer: From the definition for c as $c = (a^2 - b^2)^{1/2}$ we have c = 17.1 AU and so the perihelion distance is just d = 17.8 – 17.1 = 0.7 AU. Since 1 AU = 150 million km, it comes to within **105 million km** of the sun. This is near the orbit of Venus.