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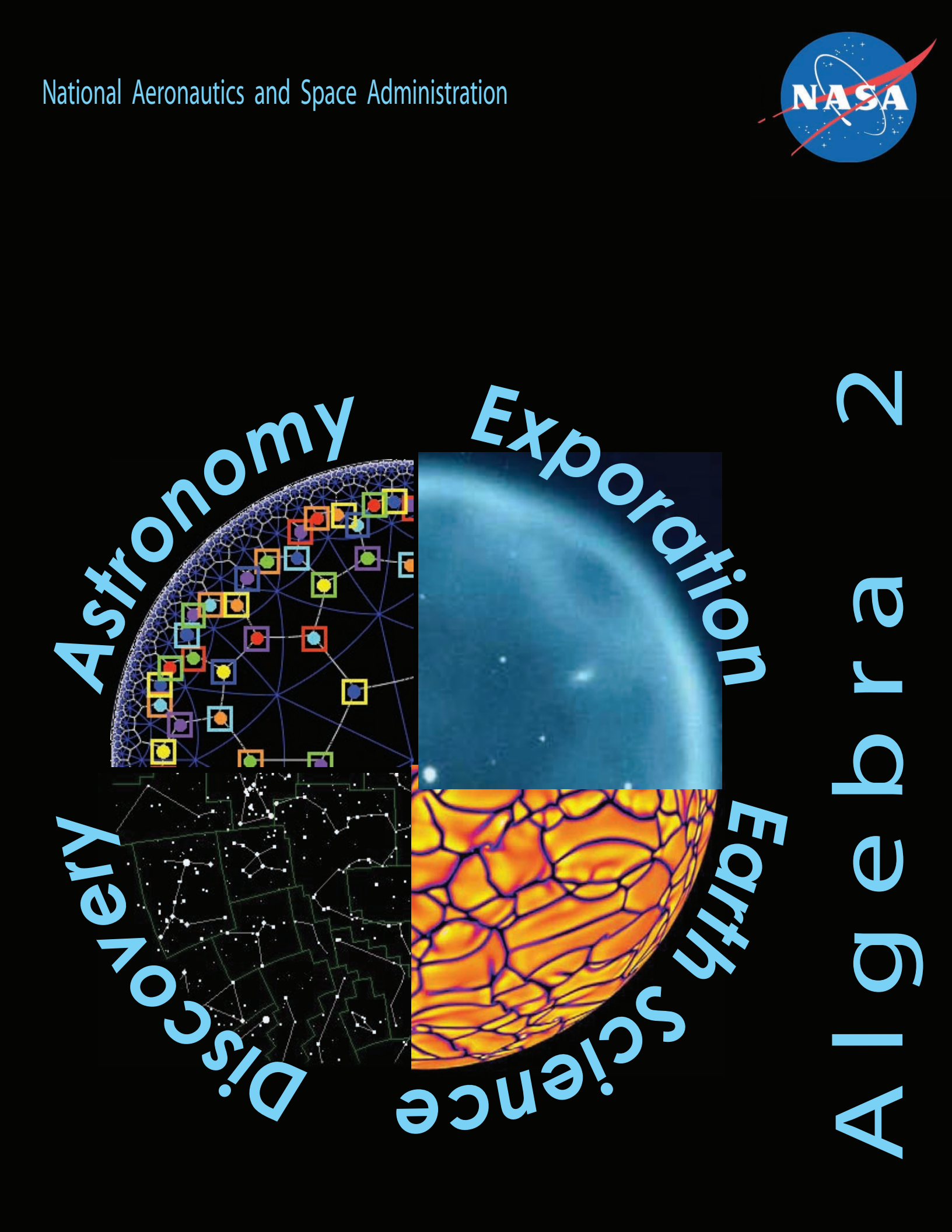

Algebra 2

Algebra 2

A supplementary collection of math problems featuring astronomy and space science applications

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Introduction

The two-year investigation by the National Mathematics Advisory Panel examined more than 16,000 research publications and policy reports. The importance of algebra was emphasized in the report because, as the panel reported, “The sharp falloff in mathematics achievement in the U.S. begins as students reach late middle school, where, for more and more students, algebra course work begins” (p. xiii). The report found that “to prepare students for algebra, the curriculum must simultaneously develop conceptual understanding, computational fluency and problem-solving skills.” Further, it said: “Debates regarding the relative importance of these aspects of mathematical knowledge are misguided. These capabilities are mutually supportive.”

Due to the poor performance of U.S students on international assessments such as the Trends in International Mathematics and Science Study (TIMSS) and the OECD Programme for International Student Assessment (PISA), the entire U.S approach to mathematics education has come into question. Integrating mathematics with other subjects is now being revisited as an approiach that may make a difference. A number of national science and mathematics education professional associations are are united in their support for the integration of science and mathematics teaching and learning. For instance, documents published by the following associations all recommend a more aggreessive integration of science and mathematics education: American Association for the Advancement of Science; National Council of Teachers of Mathematics; National Research Council; and the National Science Teachers Association. The current evolution in thinking is now that mathematics experiences should allow students to learn about mathematics by working on problems arising in contexts outside of mathematics. These connections can be to other subject areas and disciplines as well as to students' daily lives.

The integration of mathematics into other subject areas is not a new concept and has been around for over a century. The chief advantages are that it allows students to see how many of our ‘real world’ decissions are based upon some type

of mathematical understanding, whether it involves low-level skills of figuring tips, splitting up a restaurant bill among friends, or compound interest and stock trades. Above all, it also helps answer the common student question “When are we ever going to use this?” But there are many other reasons why mathematics integration is critically important.

From advances in brain research over the last 50 years, we know that the human brain looks for patterns and interconnections as its way of making sense of things. This is not usually the way in which mathematics and physical science are taught. As a scientist, it is inconceivable that one would consider explaining concepts in science without ever addrressing the mathematical underpinnings from which the concepts are derived, and are manifestly integrated into a logical framework. Teaching science as an extension of an english course, where only the mastery of a specialized vocabulary and the learning of ‘facts’ is important, does a complete disservice to the compelling logoical scaffolding behind scientific statements, hypothesis and theories, that cannot be accessed without also understanding their mathematical relationships.

The current education system seems to be predicated on the assumption that students will, on their own, make the associations between math and science, and will eventually see how the subjects fit together and into the real world. Without any pevious experience of seeing how this is done in grades K-8, they have no way to actually model this critical step. However, when mathematics is integrated with science, plenty of examples are available for the student to see how this critical integration process happens. Moreover, teachers do not need to guess about whether the connections have been made by students, the connections will be clear.

Algebra II is a course in mathematics offered in the United States public and private school systems taken by approximately 85% of all graduating high school seniors by the age of 17. Two major studies by the U.S Department of Education have shown that Algebra II is a ‘gateway ‘course that predicts student graduation from college, and their eventual qualification for high-paying careers. The course is typically taught in Grade 10 as a two-semester series following prerequisite courses in Algebra I and/or Geometry. The course stresses student mastery of the analysis and graphing of polynomials, logarithmic, exponential and trigonometric functions, with some applications to real-world problems in which these modeling techniques can often be seen to apply.

In keeping with the intent to show how Algebra II topics connect with real-world applications, textbooks commonly include several hundred ‘word problems’ that are generally culled from situations that students may encounter, often involving economics. What appears to be absent from the selection are an adequate number of problems in earth or space science. For example, out of 700 application problems in ‘Algebra II’ (McDougal-Littell, 2004) one finds fewer than 30 that connect with physical science or space science. Many of these are fairly generic and do not leverage recent discoveries in earth or space science as a way to ‘hook’ the student’s interest in these topics and prospective careers.

Since 2004, Space Math@ NASA has developed math problems for grades 3-12 designed to showcase how NASA discoveries in earth and space science are connected to a variety of math topics and skills. By 2010, over 340 of these problems are available online, or can be found in a series of special-topic books (Black Hole Math, Earth Math, etc). Frequently, NASA press releases serve as the ‘hook’ to provide a suitable topic from which an appropriate mathematical problem is developed. This also allows students to read about a new discovery on the ‘Evening News’ or CNN.com, and then within a few days they can work through some mathematical issue presented by the news release. For example, the Gulf Oil Spill of 2010 was viewed by the NASA,

Terra satellite and students used the satellite image to calculate its total area, mass and density. In other examples, students can read a press release announcing the discovery of a new planet, and calculate from two points on its elliptical orbit, the equation of the orbit, its semi-major axis and the orbit period of the planet.

This book contains over 200 problems spanning over 70 specific topic areas covered in a typical Algebra II course. The content areas have been extracted from the McDougal-Littell 'Algebra II' textbook according to the sequence used therein. A selection of application problems featuring astronomy, earth science and space exploration were then designed to support each specific topic, often with more than one example in a specific category. Each problem is introduced with a brief paragraph about the underlying science, written in a simplified, non-technical jargon where possible. Problems are often presented as a multi-step or multi-part activities. The intent of these problems is not to follow an explicitly 'inquiry-based' approach, but to systematically show students how problems and questions of a specific type are often solved. Once students have mastered a particular approach, there are many opportunities available for students to 'go beyond' each problem and inquire about other connections that may suggest themselves as the student completes each problem, or a collection of problems.

This book is not a replacement for standard Algebra II textbooks. It does not provide any pedagogical information about how to 'teach' a particular topic area. Instead, this book is a supplementary resource that the teacher may use to increase the number of applications problems at their disposal for teaching the specific topics. The problems may be used as-is, adapted, or shortened depending on the needs of the particular student or classroom situation. Teachers and students are encouraged to visit the Space Math @ NASA website to download the latest math problems spanning many other math topic areas, which may work for math remediation, or preparation for Algebra II concepts.

Alignment with Standards

This book was patterned after an extant textbook, 'Algebra II' in scope and sequence. Consequently, the selection of problems and their sequence through the book parallel the development and motivational arguments made by the publisher, McDougal-Littell, in their compliance with state and national mathematics standards of learning.

AAAS Project:2061 Benchmarks

(9-12) - Mathematical modeling aids in technological design by simulating how a proposed system might behave.

2B/H1 ---- Mathematics provides a precise language to describe objects and events and the relationships among them. In addition, mathematics provides tools for solving problems, analyzing data, and making logical arguments.

2B/H3 ----- Much of the work of mathematicians involves a modeling cycle, consisting of three steps: (1) using abstractions to represent things or ideas, (2) manipulating the abstractions according to some logical rules, and (3) checking how well the results match the original things or ideas. The actual thinking need not follow this order. 2C/H2

Teacher Comments

"Your problems are great fillers as well as sources of interesting questions. I have even given one or two of your problems on a test! You certainly have made the problems a valuable resource!" (Chugiak High School, Alaska)

"I love your problems, and thanks so much for offering them! I have used them for two years, and not only do I love the images, but the content and level of questioning is so appropriate for my high school students, they love it too. I have shared them with our math and science teachers, and they have told me that their students like how they apply what is being taught in their classes to real problems that professionals work on." (Wade Hampton High School ,SC)

"I recently found the Space Math problems website and I must tell you it is wonderful! I teach 8th grade science and this is a blessed resource for me. We do a lot of math and I love how you have taken real information and created reinforcing problems with them. I have shared the website with many of my middle and high school colleagues and we are all so excited. The skills summary allows any of us to skim the listing and know exactly what would work for our classes and what will not. I cannot thank you enough. I know that the science teachers I work with and I love the graphing and conversion questions. The "Are U Nuts" conversion worksheet was wonderful! One student told me that it took doing that activity (using the unusual units) for her to finally understand the conversion process completely. Thank you!" (Saint Mary's Hall MS, Texas)

"I know I'm not your usual clientele with the Space Math problems but I actually use them in a number of my physics classes. I get ideas for real-world problems from these in intro physics classes and in my astrophysics classes. I may take what you have and add calculus or whatever other complications happen, and then they see something other than "Consider a particle of mass 'm' and speed 'v' that…" (Associate Professor of Physics)

"Space Math has more up-to-date applications than are found in any textbook. Students enjoy real-world math problems for the math they have already learned. Doing Space Math problems has encouraged some of my students to take pre-calculus and calculus so they can solve the more advanced problems. I learned about Space Math through an email last year. I was very impressed with the problems. I assigned some of the problems to students in my Physics classes, printing them out to put in their interactive notebooks. I displayed other problems for group discussion, assigned some for homework and used some for group class work. I like the diversity, the color format and having the solutions. I expect to use them even more next year in our new space science class. We will have 50 students in two sections." (Alan. High School Science Teacher)

"It took time for them to make the connection between the math they learned in math class and applying it in the science classroom. Now I use an ELMO to project them. I have used them for class work and/or homework. The math activities were in conjunction with labs and science concepts that were being presented. The math helped "show" the science. Oftentimes students were encouraged to help and teach each other. Students began to see how math and science were connected. I knew the students were making the connections because they would comment about how much math they had to do in science. Their confidence in both classes increased as they were able practice the concepts they learned in math in my science class." (Brenda, Technology Resource Teacher)