National Aeronautics and Space Administration

Exploring Aerosols

SAGE III Math

This collection of activities is intended for students looking for additional challenges in the math and physical science curriculum in grades 6 through 12. The problems were created to be authentic glimpses of modern science and engineering issues, often involving actual research data.

The problems were designed to be ‘one-pagers’ with a Teacher’s Guide and Answer Key as a second page. This compact form was deemed very popular by participating teachers.

For more weekly classroom activities about astronomy and space visit the NASA website,

http://spacemath.gsfc.nasa.gov

Add your email address to our mailing list by contacting Dr. Sten Odenwald at

Sten.F.Odenwald@nasa.gov

Image Credits: Front: On Saturday, Aug. 27, 2011, International Space Station astronaut Ron Garan used a high definition camera to film one of the sixteen sunrises astronauts see each day. This image shows the rising sun as the station flew along a path between Rio de Janeiro, Brazil and Buenos Aires, Argentina

Back: High-resolution global atmospheric modeling run on the Discover supercomputer at the NASA Center for Climate Simulation at Goddard Space Flight Center, Greenbelt, Md., provides a unique tool to study the role of weather in Earth's climate system. The Goddard Earth Observing System Model, Version 5 (GEOS-5) is capable of simulating worldwide weather at resolutions of 10 to 3.5 kilometers (km). This portrait of global aerosols was produced by a GEOS-5 simulation at a 10-kilometer resolution. Dust (red) is lifted from the surface, sea salt (blue) swirls inside cyclones, smoke (green) rises from fires, and sulfate particles (white) stream from volcanoes and fossil fuel emissions. Image credit: William Putman, NASA/Goddard

This booklet was created through an education grant NNH06ZDA001N-EPO from NASA's Science Mission Directorate, and from a grant from the SAGE-III Education and Public Outreach Program.

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Mathematics Topic Matrix

Topic	Problem Numbers																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	L1	L2
Inquiry																			X	X
Technology, rulers																			X	X
Numbers, patterns, percentages	X		X		X	X		X	X										X	X
Averages																				
Time, distance, speed, density		X		X						X	X									
Areas and volumes																				
Scale Drawings, proportions				X									X			X			X	X
Geometry																X			X	X
Scientific Notation		X							X		X	X		X			X			
Unit Conversions		X			X				X	X	X	X	X	X			X			
Fractions		X																		
Graph or Table Analysis	X					X	X	X				X								
Solving for X														X	X					
Evaluating Fns																				
Modeling							X							X					X	X
Probability																				
Rates/Slopes						X	X		X											
Logarithmic Fns																				
Exponential Fns														X	X		X	X		X
Polynomials																				
Power Fns															X					
Conics																				
Piecewise Fns																				X
Trigonometry																				
Integration																				
Differentiation																				
Vectors																				

Alignment with Standards

Next Generation Science Standards

MS-ESS2D- Weather and Climate

- **Performance Expectation: MS-ESS3-5.**
  Ask questions to clarify evidence of the factors that have caused the rise in global temperatures over the past century.

CCMS: Common Core Mathematics Standards

Grades 6–8

CCSS.Math.Content.7.RP.A.2c Represent proportional relationships by equations

CCSS.Math.Content.6.EE.A.2c Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems

CCSS.Math.Content.6.SP.B.4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots

CCSS.Math.Content.7.G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

CCSS.Math.Content.8.EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used.

CCSS.Math.Content.8.EE.C.7b Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

CCSS.Math.Content.8.EE.C.8c Solve real-world and mathematical problems leading to two linear equations in two variables

Grades 9–12

CCSS.Math.Content.HSN-RN.A.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents

CCSS.Math.Content.HSN-Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

CCSS.Math.Content.HSA-SSE.B.3c Use the properties of exponents to transform expressions for exponential functions.

CCSS.Math.Content.HSA-CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

CCSS.Math.Content.HSF-TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

An Introduction to SAGE III Science

The Third Stratospheric Aerosol and Gas Experiment (SAGE III) instrument is used to study ozone, a gas found in the upper atmosphere that acts as Earth’s sunscreen. More than 25 years ago, scientists realized there was a problem with Earth’s thin, protective coat of ozone...it was thinning. The SAGE family of instruments was pivotal in making accurate measurements of the amount of ozone loss in Earth's atmosphere. SAGE instruments have also played a key role in measuring the onset of ozone recovery resulting from the internationally mandated policy changes that regulated chlorine-containing chemicals, the Montreal Protocol, which was passed in 1987.

Instead of flying on an un-manned satellite, SAGE III will be mounted to the International Space Station (ISS) where it will operate alongside experiments from all over the world in the space-based laboratory. The orbital path of ISS will help maximize the scientific value of SAGE III observations.

SAGE III - ISS is scheduled to board one of NASA's first commercial Space X flights in 2014 for a ride to its new home. Once on ISS, SAGE III will do what it does best – Earth observations to extend a long record of atmospheric measurements for the continued health of Earth and its inhabitants.

More specifically, SAGE III - ISS will provide global, long-term measurements of key components of the Earth's atmosphere. The most important of these are the vertical distribution of aerosols and ozone from the upper troposphere through the stratosphere. In addition, SAGE III also provides unique measurements of temperature in the stratosphere and mesosphere and profiles of trace gases such as water vapor and nitrogen dioxide that play significant roles in atmospheric radiative and chemical processes.

The SAGE III mission is an important part of NASA's Earth Observation System and is designed to fulfill the primary scientific objective of obtaining high quality, global measurements of key components of atmospheric composition and their long-term variability. The primary focus of SAGE III on ISS will be to study aerosols, clouds, water vapor, pressure and temperature, nitrogen dioxide, nitrogen trioxide, and chlorine dioxide.

Aerosols

Aerosols play an essential role in the radiative and chemical processes that govern the Earth's climate. Since stratospheric aerosol loading has varied by a factor of 30 since 1979, long-term monitoring of tropospheric and stratospheric aerosols is crucial. SAGE III aerosol measurements will provide important contributions in the area of aerosol research.

Clouds

Clouds play a major role in determining the planet's solar and long-wave energy balance and, thus, are important in governing the Earth's climate. SAGE III will provide measurements of mid and high level clouds including thin or "sub-visual" clouds that are not detectable by nadir-viewing passive remote sensors. These observations are important because while low clouds primarily reflect incoming solar radiation back into space (acting to cool the planet), mid and high level clouds enhance the "greenhouse" effect by trapping infrared radiation (acting to warm the planet). Also, the presence of thin cloud near the tropopause may play a significant role in heterogeneous chemical processes that lead to ozone destruction in mid-latitudes.

Water vapor

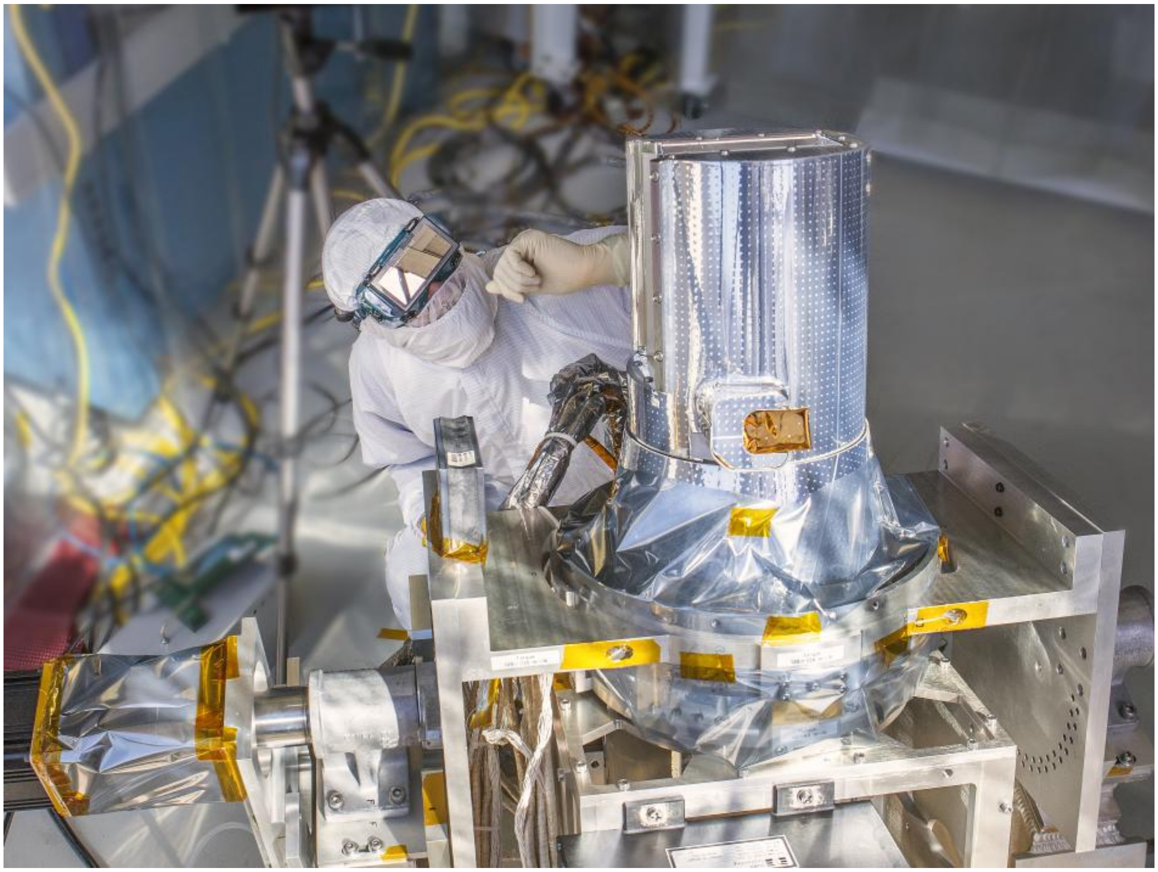
On a molecule-by-molecule basis, water vapor is the predominant greenhouse gas and plays a crucial role in regulating the global climate system. An improved understanding of the global water vapor distribution can enhance our ability to understand water's role in climate processes. SAGE III water vapor measurements will provide important contributions on the long term effect of this greenhouse gas.

Ozone

From September 21 to 30, 2006, the average area of the ozone hole was the largest ever observed, at 10.6 million square miles (27.5 million square kilometers). This image, from September 24, the Antarctic ozone hole was equal to the record single-day largest area of 11.4 million square miles (29.5 million square kilometers), reached on Sept. 9, 2000. The blue and purple colors are where there is the least ozone, and the greens, yellows, and reds are where there is more ozone.

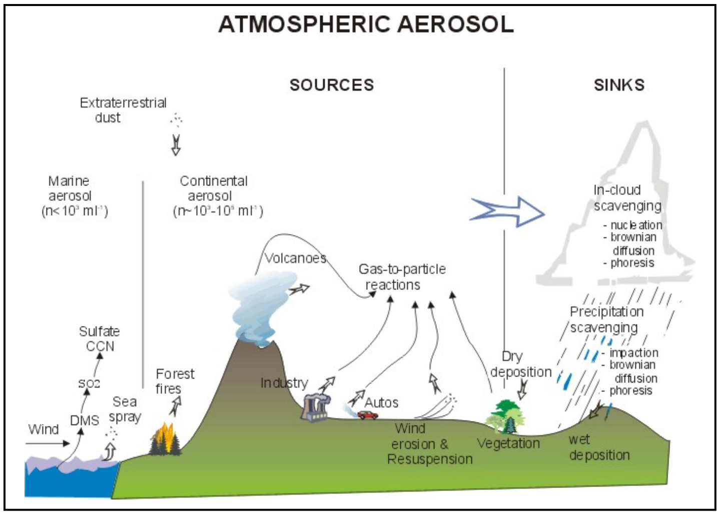
Ozone research has remained at the forefront of atmospheric science for many years because stratospheric ozone shields the Earth's surface (and its inhabitants) from harmful ultraviolet radiation. Since recent declines in stratospheric ozone have been linked to human activity, accurate long-term measurements of ozone remain crucial.

It is important to monitor ozone levels in the lower stratosphere and upper troposphere since observed trends are the largest and most poorly understood at those altitudes. SAGE III's high vertical resolution and long-term stability make it uniquely well suited to make these measurements. SAGE III will also be able to look at the relationship between aerosol, cloud, and chemical processes affecting ozone argue for simultaneous measurements of these atmospheric constituents (such as those made by SAGE III).



NASA engineer Chip Holloway waits for the sun to align with the Stratospheric Aerosol and Gas Experiment (SAGE III) during a clean room "sun-look" test on March 4, 2013, at NASA's Langley Research Center in Hampton, Va. SAGE III passed this test, successfully locking onto the sun and completing a series of measurements. Scientists are checking SAGE III in preparation for its trip to the International Space Station, set for late 2014 or early 2015. Like its predecessors SAGE I and II, which collected aerosol data from 1979 to 2005, SAGE III will measure aerosols, ozone, water vapor and other gases to help scientists better understand Earth's atmosphere.

Atmospheric Aerosols by Percentage



Atmospheric aerosols can come in all sizes and types. The origin of these aerosols is also very complicated. Over the years scientists have been able to measure the percentages of the various types from different parts of the world.

The table below shows the results in percent compiled by NOAA's Earth Systems Research Laboratory.

Atmospheric Aerosols						
Location	Sea Salt	Dust	Water Droplets	Sulfate (SO_4)	Organic Particulates (PAH, etc)	Other
Marine	40%		30%	15%		15%
Europe	3%		23%	46%	10%	18%
Africa	5%	3%	13%	40%	20%	19%
India		4%	28%	44%	5%	19%
Asia	1%	22%	20%	27%	20%	10%
USA		1%	20%	19%	50%	10%

Problem 1 – In which direction (rows or columns) do you expect the percentages to add up to 100% and why?

Problem 2 – Draw a pie graph showing the percentages of the various aerosol contributions over the United States. What type of aerosol is the most abundant?

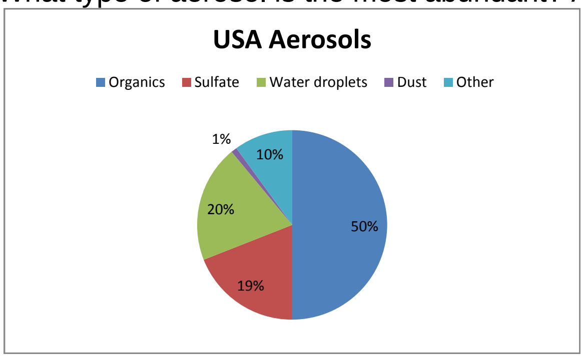
Problem 3 – Create a pie graph that shows the percentage of sulfate aerosols over the 6 different regions. Which region has the highest percentage?

Problem 4 – Sulfate aerosols and organic particulates are typically man-made from the burning of fossil fuels. Which region has the highest concentration of man-made sources of aerosols, and why do you think this might be?

NOAA aerosol data from Earth System Research Laboratory:
http://www.esrl.noaa.gov/research/themes/aerosols/#fig1

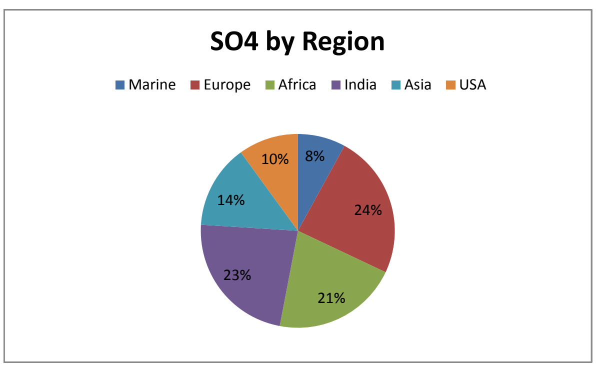
Problem 1 – In which direction (rows or columns) do you expect the percentages to add up to 100% and why? Answer: **The total percentage of aerosols has to add up to 100% because this table is indicating the types of aerosols found in the study. The only direction in which this happens in the table is along the rows, so each row adds up to 100% for the indicated location.**

Problem 2 – Draw a pie graph showing the percentages of the various aerosol contributions over the United States. What type of aerosol is the most abundant? Answer: See below.



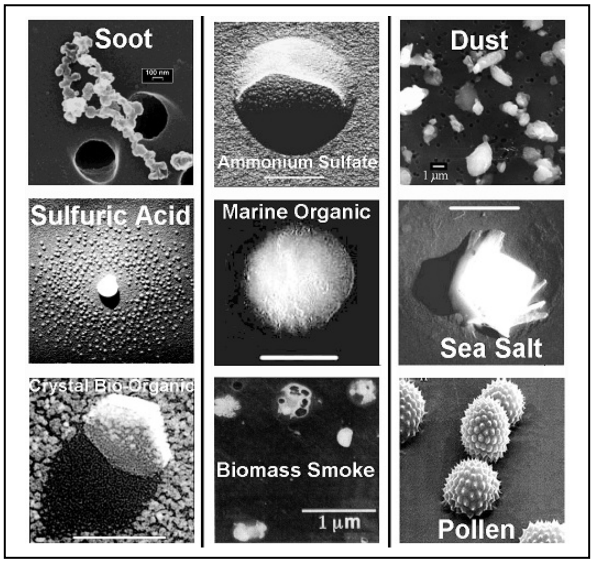
Problem 3 – Create a pie graph that shows the percentage abundance of sulfate aerosols over the 6 different regions. Which region has the highest abundance?

Answer: First you have to add up the percentages for each region: 15% + 46% + 40% + 44% + 27% + 19% = 191%. Then divide each percentage by 191% to get the percentage for each region. Marine = 100% x (15/191) = 8%; Europe = 24%; Africa = 21%; India = 23%; Asia= 14% and USA= 10%. Now create the pie graph. Europe has the highest concentration (24%).



Problem 4 – Sulfate aerosols and organic particulates are typically man-made from industrial processes and the burning of fossil fuels. Which region has the highest concentration of industrial sources of aerosols and why do you think this trend occurs?

Answer: From the table, if we combine the columns for Sulfate and Organic Particulates we see that the **USA with 69% is the largest**. This is probably because the USA is more 'industrialized' than other countries, especially along its East Coast.



An aerosol is a mixture of fine solid particles or liquid droplets in air or another gas. Examples of aerosols include clouds, haze, and air pollution such as smog and smoke. The liquid or solid particles have diameter mostly smaller than 1 μm or so.

Aerosols are so small we have no real choice but to use scientific notation to determine their properties such as volume, mass or density! The graph to the left shows the sizes of different types of aerosols in terms of nanometers, where 1 nanometer is 1 one billionth of a meter or 10^{-9} meters.

Problem 1 – A human hair has a diameter of 100 micrometers (100 microns). If 1000 nanometers equals 1 micron, how many 250 nanometer aerosol particles can fit across the diameter of one human hair?

Problem 2 – If the density of a typical spherical sea salt aerosol particle is 2.0 grams/cm^3, and the particle has a diameter of 500 nanometers, what is the mass of a single aerosol particle in A) grams? B) micrograms?

Problem 3 - On an especially hazy day, the density of aerosol particles in the air is 10 million particles per cubic centimeter. If the particles have an average size of 900 nanometers and a density of 1.5 grams/cm^3, A) how much aerosol mass would there be in a cubic meter of air? B) If you breath-in 100 liters of air every minute, and 1 liter equals 1000 cm^3, how many grams of aerosols do you inhale every day?

Problem 1 – A human hair has a diameter of 100 micrometers (100 microns). If 1000 nanometers equals 1 micron, how many 250 nanometer aerosol particles can fit across the diameter of one human hair?

Answer: Diameter of human hair in nanometers = 100 micrometers x (1000 nanometers/1 micrometer) = 100000 nanometers. Since one aerosol is 250 nm in diameter, there would be 100000/250 = **400 aerosol particles** place end to end to cross the diameter of one human hair.

Problem 2 – If the density of a typical spherical sea salt aerosol particle is 2.0 grams/cm^3, and the particle has a diameter of 500 nanometers, what is the mass of a single aerosol particle in A) grams? B) micrograms?

Answer: A) The radius of the aerosol is 250 nanometers. Since 1 meter = 100 centimeters, the radius is 250 x 10^{-9} x (100 cm/1 meter) = 2.5×10^{-5} cm. Volume of a spherical particle = 4/3 π (2.5×10^{-5} meters)^3 = 6.5×10^{-14} cm^3. The mass in grams is then 2.0 grams/cm^3 x Volume in cm^3, so **M = 1.3×10^{-13} grams**.

B) 1 microgram = 10^{-6} grams, so **M = 1.3×10^{-7} micrograms**.

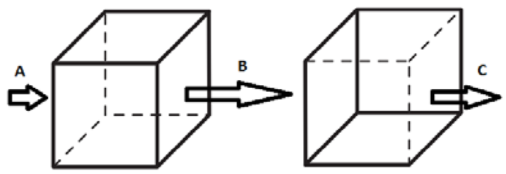
Problem 3 - On an especially hazy day, the density of aerosol particles in the air is 10 million particles per cubic centimeter. If the particles have an average size of 900 nanometers and a density of 1.5 grams/cm^3, A) how much aerosol mass would there be in a cubic meter of air? B) If you breath-in 100 liters of air every minute, and 1 liter equals 1000 cm^3, how many grams of aerosols do you inhale every day?

Answer: A) Each particle has a mass of

$$M = 1.5 \text{ gm/cm}^3 \times (4/3\ \pi\ (4.5\times10^{-5} \text{ cm}))^3 = 5.7\times10^{-13} \text{ grams.}$$

If the particle density is 10^7 particles/cm^3, in 1 cubic meter there would be 10^7particles/cm^3 x 10^6 cm^3= 10^{13} particles, and so the total mass per cubic meter would be 5.7×10^{-13} grams x 10^{13} particles = **5.7 grams**!

B) You breath in 100 liters/minute x (1000 cm^3/1 liter) x (60 minutes/1 hour) x (24 hours/1 day) = 1.44×10^8 cm^3. Since the aerosol mass is 5.7 grams/meter^3 we have 1.44×10^8 cm^3 x (1 meter^3/10^6 cm^3) x (5.7 grams/1 meter^3) = 144 x 5.7 = **821 grams/day**.



A ray of light passes through the two cubes from left to right. At **A**, the light ray has its full intensity of 100%.

After it leaves the first box, the aerosols have reduced the light intensity by 10% so that at B, the light intensity is 100% x (0.90) = 90%.

After passing through an identical aerosol region in the second box, the light intensity is reduced by an additional 10%. The final light intensity at C is then 100% x (0.90) x (0.90) = 81%.

Aerosols are small particles of liquids or solids that are light enough to be suspended in the air for long periods of time. Common aerosols can include dust from volcanoes, exhaust from jet plains, smog, or ash from the combustion of fossil fuels and wood. Most of the time you do not even notice they are there, unless they are present in large enough numbers.

When the concentration of aerosols is high enough, they can actually cause the dimming of sunlight, which is why the sky can appear darker on a foggy day (water aerosols) or very hazy on a day with lots of smog or distant forest fires.

The figure to the left shows how light dimming occurs, if the intensity of light is reduced by 10% as it passes through two boxes of air.

Problem 1 – Suppose in the above example, the light ray passes through 6 identical boxes. How bright will the light be after it leaves the sixth box?

Problem 2 – Suppose that each box is 1 kilometer on a side, and that the light is dimmed by1% through each box. If the total path is 20 kilometers, what will be the brightness of the light after it leaves this aerosol cloud to the nearest percent?

Problem 3 – A light ray passes through 5 kilometers of ordinary air, which reduces the light by 0.5% per kilometer, and then passes through a dense cloud that reduces the light by 5% per kilometer. If the cloud is 3 kilometers long, to the nearest percentage what will be the light intensity when the light leaves the cloud?

Problem 1 – Suppose in the above example, the light ray passes through 6 identical boxes. How bright will the light be after it leaves the sixth box?

Answer: 100% x 0.9 x 0.9 x 0.9 x 0.9 x 0.9 x 0.9 = 100% x $(0.9)^6$ = **53%**.

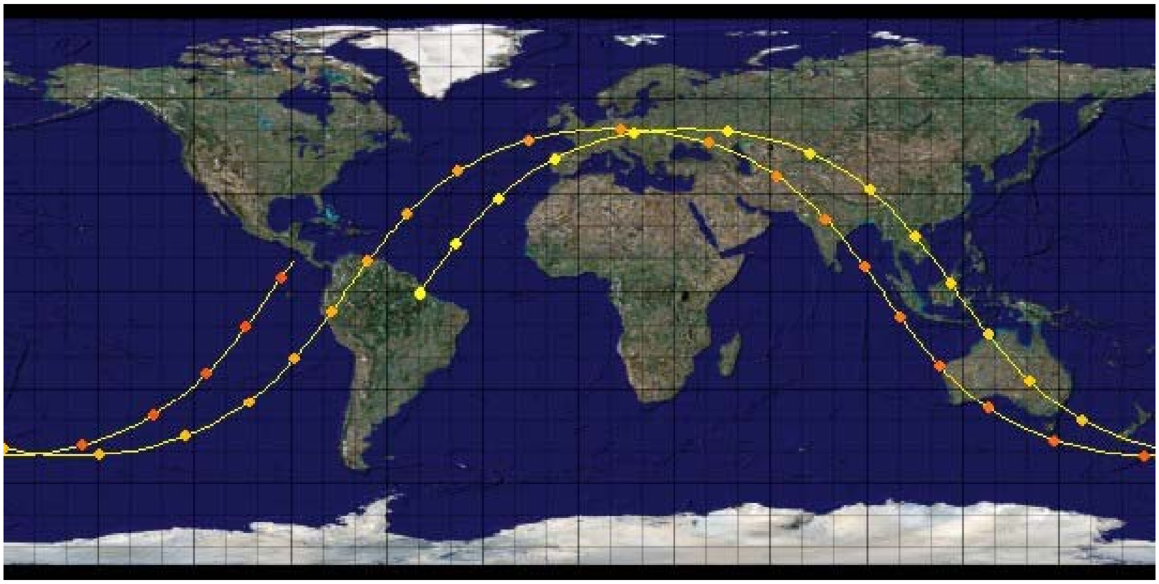
Problem 2 – Suppose that each box is 1 kilometer on a side, and that the light is dimmed by1% through each box. If the total path is 20 kilometers, what will be the brightness of the light after it leaves this aerosol cloud to the nearest percent?

Answer: There are 20 boxes along the light ray so 100% x $(0.99)^{20}$ = **82%**.

Problem 3 – A light ray passes through 5 kilometers of ordinary air, which reduces the light by 0.5% per kilometer, and then passes through a dense cloud that reduces the light by 5% per kilometer. If the cloud is 3 kilometers long, to the nearest percentage what will be the light intensity when the light leaves the cloud?

Answer: 100% x $(0.995)^5$ x $(0.95)^3$ = 100% x 0.975 x 0.857 = **84%**

Visibility and dimming: Typical morning fogs can attenuate light by 50% but there is still enough light to read a book, it's just that the light passing through the fog is scattered, which means that you cannot see an object clearly if it is more than a few hundred meters away.



As the ISS orbits Earth every 90 minutes, Earth rotates ‘underneath’ the orbit of the ISS once every 24 hours. This means that each ISS orbit advances in longitude by a fixed amount in longitude. The figure above shows the ground track of the ISS for two orbits. Each square is 10 degrees on a side, with the Equator running horizontally across the middle of the diagram. The squares are shown at intervals of 5 minutes.

Problem 1 - If Earth rotates 360 degrees in 24 hours, by how many degrees does it shift during the orbit of the ISS?

Problem 2 – How many sunsets and sunrises will the SAGE-III instrument on the ISS observe each 24-hour day?

Problem 3 – How many orbits will it take before the ISS passes over the same spot on the Equator?

Problem 4 - During each orbit, the ISS will cross the Equator traveling north to south, then after ½ orbit (45 minutes) it will cross the Equator traveling south to north. How long will you wait before you see the ISS from the ground traveling north-to-south across the sky directly overhead?

Problem 1 - If Earth rotates 360 degrees in 24 hours, by how many degrees does it shift during the orbit if the ISS?

Answer: 360/24 = X/1.5 so **X = 22.5 degrees**.

Problem 2 – How many sunsets and sunrises will the SAGE-III instrument on the ISS observe each 24-hour day?

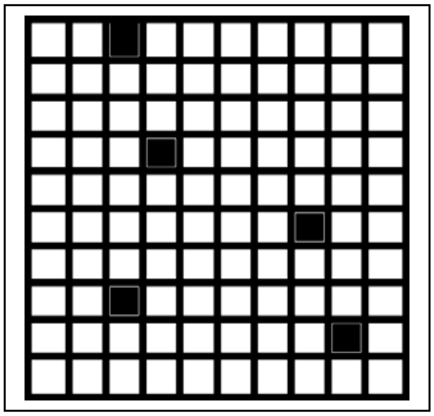
Answer: 24 h /1.5 h = **16 sunrises, and an equal number of sunsets** for a total of 32 events. A sunrise is followed by a sunset every 90/2 = 45 minutes!

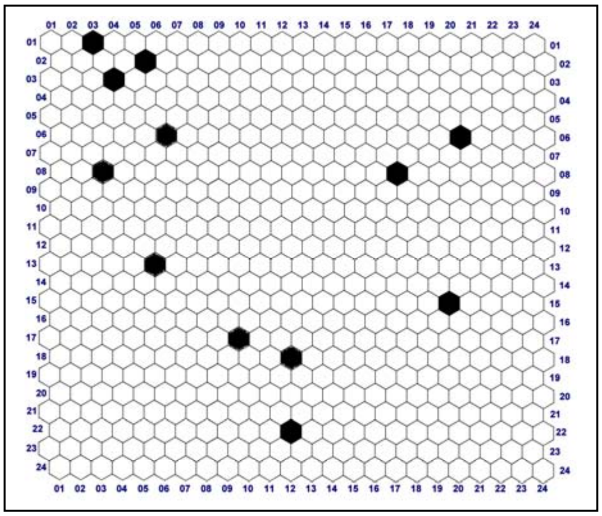
Problem 3 – How many orbits will it take before the ISS passes over the same spot on the Equator?

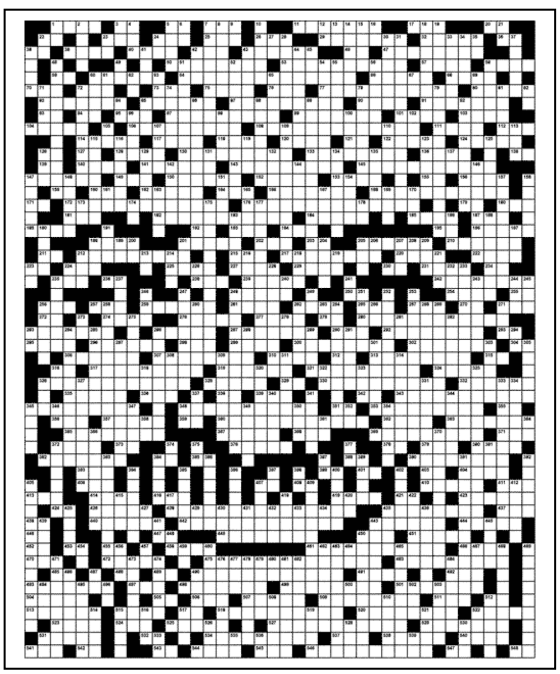
Answer: The orbit advances 22.5 degrees in longitude every orbit, so it will take 360/22.5 = 16 orbits or one full day to return to the same longitude. However, because the ISS crosses the equator twice every orbit, it actually takes only **8 orbits or 12 hours**.

Problem 4 - During each orbit, the ISS will cross the Equator traveling north to south, then after ½ orbit (45 minutes) it will cross the Equator traveling south to north. How long will you wait before you see the ISS from the ground traveling north-to-south across the sky directly overhead?

Answer: Every **16 orbits** as seen from the ground, the ISS is traveling in the same direction, so you will have to wait **24 hours**.







The Stratospheric Aerosol and Gas Experiment III (SAGE III), the sensor will be installed on the International Space Station (ISS) sometime in 2014.

Aerosols are small particles that are suspended in the air. Examples include fog and smog, but also includes dust, soot and ash particles. These particles can affect climate, and can also cause health problems such as emphysema, asthma or even lung cancer.

Instead of measuring the amount of aerosols or pollutants by percentage, scientists often use units such as parts-per-million. This exercise helps you work with these units.

Problem 1 - Suppose you are 15 years old. How many parts-per-hundred is this of one century?

Problem 2 – You have a bag of 200 blue marbles, 280 green marbles and 20 red marbles. How many parts-per-thousand are the red marbles compared to the whole?

Problem 3 – Which of the figures to the left shows a concentration of black spots equal to

A) 243000 parts-per-million?

B) 50000 parts-per-million?

C) 20833 parts per million?

Problem 1 - Suppose you are 15 years old. How many parts-per-hundred is this of one century?

Answer: 15 years is 15/100 = **15 pph** of 1 century.

Problem 2 – You have a bag of 200 blue marbles, 280 green marbles and 20 red marbles. How many parts-per-thousand are the red marbles compared to the whole?

Answer: The total number of marbles is 200+280+20 = 500. So the 20 red marbles is 20/500 = 4/100 or **4 pph** of the total number of marbles.

Problem 3 – Which of the figures to the left shows a concentration of black spots equal to A) 243000 parts-per-million? B) 50000 parts-per-million? C) 20833 parts per million?

Answer: First let's find out how many squares we have.

Top = 10x10 = 100 total and 5 black so 5/100 are black

Middle = 24 x 24 = 576 squares and there are 12 black so 12/576 are black

Bottom = 40 x 50 = 2000 squares and there are 486 black so 486/2000 are black

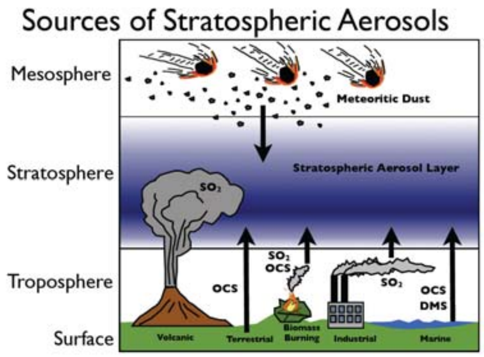
Lets use pph = parts-per-hundred
ppt = parts per thousand and
ppm = parts per million.

We can write the top as 5 pph = 50 ppt = 50,000 ppm
Middle as 12/576 = 0.0208333 = 2.0833 pph = 20.833 ppt = 20833 ppm
Bottom as 486/2000 = 0.243 = 24.3 pph = 243 ppt = 243000 ppm

So the answers are

A) Is the bottom figure
B) Is the top figure
C) Is the middle figure

Note: The concentration of carbon dioxide in our atmosphere has increased from 335 ppm to nearly 390 ppm since 1975!



(Credit: NOAA)

OCS = Carbonyl sulfide,
DMS = Dimethyl sulfide,
SO_2 = Sulfur dioxide,
CO2 = carbon dioxide.

SAGE-III is designed to measure the concentration of aerosols in the stratosphere, but where do these particles come from?

The figure to the left shows some of the common sources. Scientists are concerned about recent increases in stratospheric aerosols because they have an impact on climate change. In the stratosphere, miles above Earth's surface, aerosols can reflect sunlight back into space, which leads to a cooling influence at the ground. According to recent studies, this cooling effect may explain the changes in the pace of global warming measured since 2000.

According to recent measurements, carbonyl sulfide is produced at the following rates given in terms of millions of tons per year (Mt/yr): Marine = 0.33 Mt/yr; Volcanism = 0.05 Mt/yr; Terrestrial = 0.02 Mt/yr; Biomass Burning =0.07 Mt/yr; Industrial= 0.33 Mt/yr.

Sulfur dioxide is produced at a rate of 10 Mt/yr from volcanos; Industrial = 146 Mt/yr; Biomass burning = 8 Mt/yr.

Problem 1 – What is the total rate of production of carbonyl sulfide by all sources?

Problem 2 – What is the percentage of each source of carbonyl sulfide compared to the total production rate?

Problem 3 – Draw a circle (pie) graph that illustrates the percentages of each carbonyl sulfide source compared to the total rate.

Problem 4 – Comparing carbonyl sulfide and sulfur dioxide, which of the two has the largest percentage contribution due to human activity?

OCS rates from C. Brühl1, J. Lelieveld1,3, P. J. Crutzen1, and H. Tost2 'The role of carbonyl sulphide as a source of stratospheric sulphate aerosol and its impact on climate', Atmos. Chem. Phys., 12, 1239-1253, 2012, http://www.atmos-chem-phys.net/12/1239/2012/acp-12-1239-2012.html'

According to recent measurements, carbonyl sulfide is produced at the following rates given in terms of millions of tons per year: Marine = 0.33 Mt/yr; Volcanism = 0.05 Mt/yr; Terrestrial = 0.02 Mt/yr; Biomass Burning =0.07 Mt/yr; Industrial= 0.33 Mt/yr. Sulfur dioxide is produced at a rate of 10 Mt/yr from volcanos; Industrial = 146 Mt/yr; Biomass burning = 8 Mt/yr.

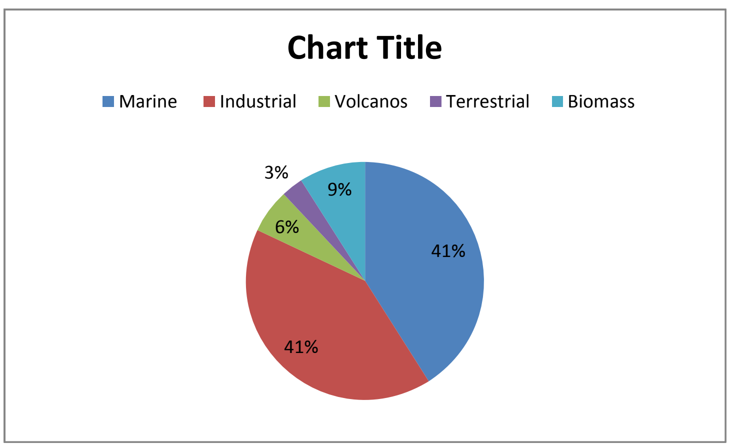
Problem 1 – What is the total rate of production of carbonyl sulfide by all sources?

Answer: 0.33 + 0.05 + 0.02 + 0.07 + 0.33 = **0.80 Mt/yr**.

Problem 2 – What is the percentage of each source of carbonyl sulfide compared to the total production rate?

Answer: **Marine: 41% Volcanism: 6% Terrestrial: 3% Biomass: 9% Industrial: 41%**

Problem 3 – Draw a circle (pie) graph that illustrates the percentages of each source compared to the total rate. Answer: See below.



Problem 4 – Comparing carbonyl sulfide and sulfur dioxide, which of the two has the largest percentage contribution due to human activity?

Answer: Total production = 164 Mt/yr. Percentages: Volcanism: 6%, Biomass burning: 5%; Industrial: 89%. **The production of sulfur dioxide by human activity is the highest: 89% vs 41%.**

Altitude (km)	Latitude		
	70N	30N	0
6	13.0	6.2	3.4
7	10.5	6.1	4.0
8	9.6	5.3	2.8
9	8.7	4.9	2.0
10	7.4	4.9	2.0
11	6.3	3.9	2.1
12	5.3	3.3	1.8
13	4.6	2.6	1.8
14	4.0	2.3	1.8
15	3.5	2.2	1.8
16	3.0	2.2	1.6
17	2.4	2.4	1.7
18	1.8	2.7	2.1
19	1.2	2.7	2.8
20	0.8	2.2	3.2
21	0.5	1.6	3.1
22	0.4	0.9	2.7
23	0.3	0.6	2.2
24	0.2	0.4	1.7
25	0.2	0.3	1.5

The SAGE-II experiment was flown on the Earth Radiation Budget Satellite beginning October 5, 1984 and ended its investigations on August 26, 2005. The SAGE-III instrument will be launched in 2014 and complete the observation program begun by the SAGE-I and SAGE-II instruments.

The SAGE instruments produce data tables like the one shown to the left. The table gives the altitude of the measurement, and subsequent columns give the extinction measured at different latitudes indicated in the top row.

The numbers indicate the aerosol extinction in units of 0.0001 km^{-1}. For example, at 6 kilometers altitude at a latitude of 30N, the extinction was 6.2 x 0.0001 = 0.00062 km^{-1}.

Problem 1 – At what location is the extinction A) the highest? B) the lowest?

Problem 2 – Above an altitude of 20 kilometers, what is the average extinction at each latitude?

Problem 3 – Graph the extinction data for 30N between an altitude of 6 and 14 km and draw a straight line through the data. What is the slope of this line, and what are the units for this slope?

Problem 4 – Write the equation of the line that you drew in Problem 3, and use it to estimate the extinction at an altitude of 19 kilometers. Is our mathematical model a good fit to the actual data?

The SAGE-II Lesson Plan for graphing data can be found at
http://science-edu.larc.nasa.gov/EDDOCS/Aerosols/aer_sci.html

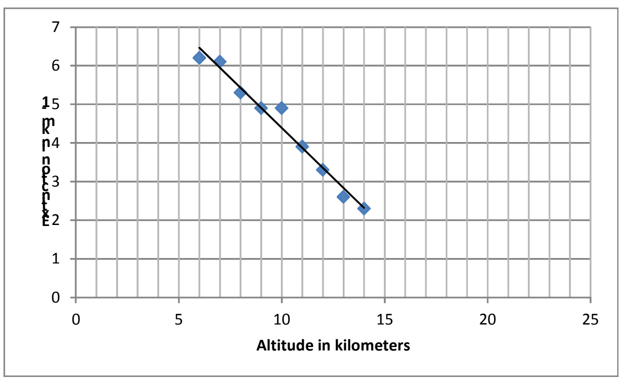
Problem 1 – At what location is the extinction A) the highest? B) the lowest?

Answer: A) **At 70N at an altitude of 6 km** above the ground where its value is 13.0 x 0.0001 = **0.0013** km^{-1}. B) **At an altitude of 24-25 km for 70N** where its value is 0.2 x 0.0001 = **0.00002 km^{-1}**.

Problem 2 – Above an altitude of 20 kilometers, what is the average extinction at each latitude?

Answer: 70N: (0.8+0.5+0.4+0.3+0.2+0.2)/6 = 0.4 or 0.4 x 0.0001 = **0.00004 km^{-1}**.
30N: (2.2+1.6+0.9+0.6+0.4+0.3)/6 = 1.0 or **0.0001 km^{-1}.**
0 N: (3.2+3.1+2.7+2.2+1.7+1.5)/6 = 2.4 or 2.4 x 0.0001 = **0.00024 km^{-1}.**

Problem 3 – Graph the extinction data for 30N between an altitude of 6 and 14 km and draw a straight line through the data. What is the slope of this line, and what are the units for this slope? Answer: See graph below. The slope is -0.52 and the units will be $(km^{-1})/km = km^{-2}$ or $1/km^2$

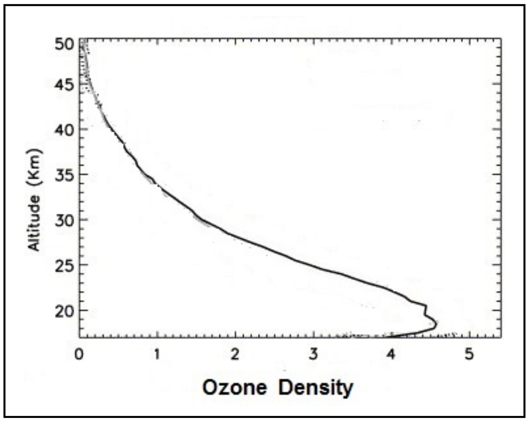


Alternate slope method using two-point formula and data table:
M = (y2-y1)/(x2-x1) so for the points at 6km and 14 km, M = (2.3-6.2)/(14 – 6) = -0.49 km^{-2}.

Students estimates should be close to M = -0.5 km^{-2}

Problem 4 – Write the equation of the line that you drew in Problem 3, and use it to estimate the extinction at an altitude of 19 kilometers. Is our mathematical model a good fit to the actual data?

Answer: The y-intercept for x=0 km is about +9.5, then **y = -0.5X + 9.5**. At an altitude of 19 km, the extinction would be y = -0.5(19)+9.5 = **0.0 km^{-1}**. The data show that the extinction is 0.00027 km^{-1} at this altitude, so we should not try to predict extinctions for altitudes outside the domain of our linear fit (6 to 14 km).



The Stratospheric Aerosol and Gas Experiment III (SAGE III), the sensor will be installed on the International Space Station (ISS) sometime in 2014. An earlier version of the SAGE-III instrument was flown in 2001 on the Russian Meteop-3M spacecraft. The new SAGE III will be using the sun and moon as light sources to measure how well the ozone layer is recovering and replenishing itself.

The ozone layer is located at an altitude of about 20 kilometers and blocks solar UV rays that would otherwise burn skin and cause cancer.

The data plot shows the density of ozone molecules at a range of altitudes as measured by the earlier SAGE-III instrument. The density of ozone molecules found in the stratosphere is presented in multiples of 1 trillion molecules per cubic centimeter.

Problem 1 – The ozone layer has the highest concentration of ozone molecules in the stratosphere. From the graph, over what altitude range does the concentration exceed 4 trillion molecules per cubic centimeter?

Problem 2 - If the density of the atmosphere in this region of the stratosphere is 400,000 trillion molecules, what fraction of the molecules are ozone if the ozone density is 4 trillion molecules/cm^3?

Problem 3 – For every million atmosphere molecules in the ozone layer, how many ozone molecules do you expect to find? (Scientists use the term parts-per-million to indicate this number.)

Data plot from
Lunar occultation with SCIAMACHY: First retrieval results
L.K. Amekudzi , A. Bracher, J. Meyer, A. Rozanov, H. Bovensmann, and J.P. Burrows
Advances in Space Research, Volume 36, Issue 5, 2005, Pages 906–914

Problem 1 – The ozone layer has the highest concentration of ozone molecules in the stratosphere. From the graph, over what altitude range does the concentration exceed 4 trillion molecules per cubic centimeter?

Answer: **Between 17 and 23 kilometers**.

Problem 2 - If the density of the atmosphere in this region of the stratosphere is 400,000 trillion molecules, what fraction of the molecules are ozone if the ozone density is 4 trillion molecules/cm^3?

Answer: 4 trillion ozone molecules/400,000 trillion atmosphere molecules = **1/100,000**

Problem 3 – For every million atmosphere molecules in the ozone layer, how many ozone molecules do you expect to find? (Scientists use the term parts-per-million to indicate this number.)

Answer: The ozone molecules are 1/100000 of the atmosphere molecules, so for every million atmosphere molecules, 1/100000 are ozone and so **10 ozone molecules** should be found for every 1 million atmosphere molecules.

This is written as 10 parts-per-million or 10 ppm.

The Sources and Sinks of Carbonyl Sulfide

Sources and sinks of carbonyl sulfide

Source	Rate (Mt/yr)
Open ocean	**+0.10**
Coastal ocean, salt marshes	**+0.20**
Anoxic soils	**+0.02**
Wetlands	**+0.03**
Volcanism	**+0.05**
Precipitation	**+0.13**
DMS oxidation	**+0.17**
Anthropogenic CS2 oxidation	**+0.21**
Natural CS2 oxidation	**+0.21**
Biomass burning	**+0.07**
Anthropogenic production	**+0.12**
Oxic soils	**-0.92**
Vegetation	**-0.56**
Reactions with OH	**-0.24**
Reactions with oxygen	**-0.02**
Photodissociation	**-0.05**

The important source of aerosols in the stratosphere (altitude from 11 to 50 km) is the formation of carbonyl sulfide (COS) droplets. Although volcanism injects millions of tons of SO_2 into the atmosphere every few years, scientists have found that during other times, COS is by far the biggest source of sulfur compounds in the stratosphere, leading to the production of sulphuric acid aerosols. Just 1 kilogram of COS is over 720 times more damaging than the same amount of carbon dioxide in altering global climate.

The table to the left gives the known sources and sinks of COS in terms of millions of tons per year (Mt/yr).

Problem 1 - In the table, sources of COS are indicated by positive rates, and systems that remove COS from the atmosphere, called sinks, are indicated by negative rates. What are the total rates for the sources and sinks, and what is the net change in atmospheric COS in megatons/year?

Problem 2 – What percentage of the sources for COS are related to human activity (anthropogenic) according to the data?

Problem 3 – 6.0×10^{23} molecules of COS has a mass of 60 grams, and 1 year equals 3.1×10^{7} seconds. What is the net change each year in the number of COS molecules in one cubic meter of the atmosphere if the volume of the atmosphere is about 4.23 billion cubic kilometers?

COS data from 'The role of carbonyl sulphide as a source of stratospheric sulphate aerosol and its impact on climate', C. Br¨uhl, J. Lelieveld, P. J. Crutzen, and H. Tost Journal of Atmospheric Chemistry and Physics, v.12, pp. 1239–1253, 2012 www.atmos-chem-phys.net/12/1239/2012/doi:10.5194/acp-12-1239-2012

Problem 1 - In the table, sources of COS are indicated by positive rates, and systems that remove COS from the atmosphere, called sinks, are indicated by negative rates. What are the total rates for the sources and sinks, and what is the net change in atmospheric COS in megatons/year?

Answer: **Total sources = +1.31 Mt/yr. Total sinks = -1.79 Mt/yr. The net change is the sum of the sources and sinks, or -0.48 Mt/yr**. This means that COS is being reduced in concentration each year at the current known rates.

Problem 2 – What percentage of the sources for COS are related to human activity (anthropogenic) according to the data?

Answer: Anthropogenic CS2 oxidation from wood burned in stoves, and direct production of this compound account for +0.21 and +0.12 Mt/year or a total of +0.33 Mt/yr. The total production is +1.31 Mt/yr, so anthropogenic sources are 100% x (0.33/1.31) = **25% of all sources.**

Problem 3 – 6.0×10^{23} molecules of COS has a mass of 60 grams, and 1 year equals 3.1×10^{7} seconds. What is the net change each year in the number of COS molecules in one cubic meter of the atmosphere if the volume of the atmosphere is about 4.23 billion cubic kilometers?

Answer: From Problem 1, the net change is a reduction by 0.48 Mt/yr.

Net reduction in tons = 480,000 tons / year x (1 year) = 480,000 tons.
Number of molecules:

$$480{,}000 \text{ tons} \times \frac{1000 \text{ kilograms}}{1 \text{ ton}} \times \frac{1000 \text{ grams}}{1 \text{ kilogram}} \times \frac{6.0 \times 10^{23} \text{ molecules}}{60 \text{ grams}} = 4.8 \times 10^{33} \text{ molecules}$$

Volume of atmosphere in cubic meters

$$= 4.23 \times 10^{9} \text{ km}^3 \times \frac{1.0 \times 10^{9} \text{ m}^3}{1 \text{ km}^3} = 4.23 \times 10^{18} \text{ m}^3$$

So: $4.8 \times 10^{33} / 4.23 \times 10^{18} \text{ m}^3 =$ **1.1×10^{15} molecules removed each year.**

A Scale Model of Aerosol Sizes

An aerosol, or 'aero-solution', is a microscopic particle made from numerous atoms and molecules stuck together. They are usually produced from chemical reactions, and the burning of organic materials like wood and hydrocarbon fuels.

The table to the left shows some common aerosol sizes along with the sizes of other objects you may know about.

Table of particle sizes

Type	Size
Atmospheric aerosol	0.015 microns
Volcanic aerosol	0.5 microns
Gasolene engine ash	20 nanometers
Diesel ash (small)	50 nanometers
Diesel ash (large)	0.4 microns
Smallpox virus	300 nanometers
E Coli bacterium	2.0 microns
Common cold virus	30 nanometers
Smog (small)	8 nanometers
Smog (large)	200 nanometers

Problem 1 – if 1 micron=1000 nanometers, order the particles by increasing size.

Problem 2 - Create a scaled model showing the relative sizes of each type of particle so that 1 nanometer = 1 millimeter in your model.

Problem 3 – A red blood cell has a diameter of 10 microns. How many volcanic aerosol particles can you place side-by-side to span this diameter?

Problem 4 – How many atmospheric aerosol particles would span the width of an e. coli bacterium?

Problem 5 – Suppose that an aerosol particle were shaped like a cube. How many atmospheric aerosol particles could you fit inside the volume of a single large particle of smog?

Problem 1 – if 1 micron=1000 nanometers, order the particles by increasing size.

Type	Size (nanometers)	Scaled Size
Smog (small)	8	8 mm
Atmospheric aerosol	15	15 mm
Gasolene engine ash	20	20 mm
Common cold virus	30	30 mm
Diesel ash (small)	50	50 mm
Smog (large)	200	20 cm
Smallpox virus	300	30 cm
Diesel ash (large)	400	40 cm
Volcanic aerosol	500	50 cm
E Coli bacterium	2000	2 meters

Problem 2 - Create a scaled model showing the relative sizes of each type of particle so that 1 nanometer = 1 millimeter in your model. Answer: See table above. Students can draw circles for the objects less than 1 meter in diameter.

Problem 3 – A red blood cell has a diameter of 10 microns. How many volcanic aerosol particles can you place side-by-side to span this diameter?

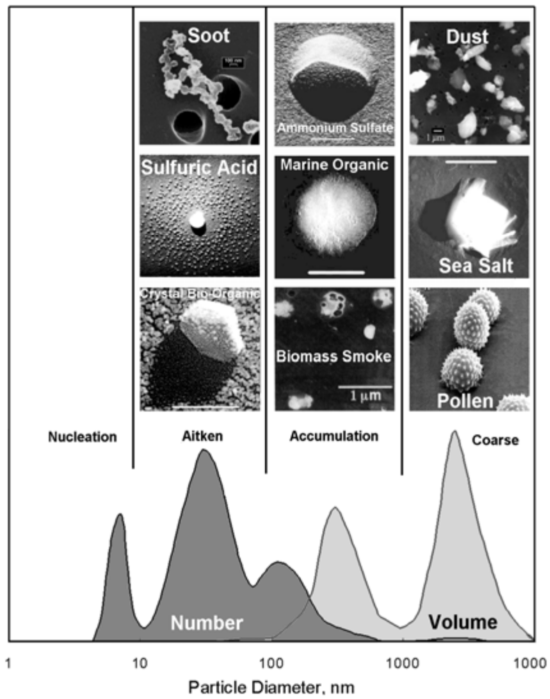
Answer: 10 microns / 0.5 microns = **20 volcanic aerosol particles**.

Problem 4 – How many atmospheric aerosol particles would span the width of an e. coli bacterium?

Answer: 2000 nanometers / 15 nanometers = **133 aerosol particles**.

Problem 5 – Suppose that an aerosol particle were shaped like a cube. How many atmospheric aerosol particles could you fit inside the volume of a single large particle of smog?

Answer: (Size of smog particle / size of aerosol)^3 = (200/15)^3 = **2370 particles**!



Aerosols are a complex ingredient to the atmosphere, which can result in both environmental and human health problems when their concentrations are too high. Smog and soot from burning fossil fuels or other organic materials can cause breathing difficulties, and perhaps even some forms of cancer.

The figure shows some common types of aerosol particles and their sizes. Most appear to be small spherical particles when seen under the microscope.

The formula for the volume of a sphere is given by $V = 4/3\ \pi\ R^3$. Also, 1 micron (μm)= 0.0001 centimeters, and 1 nanometer (nm) = 0.001 microns.

Scientists like to use two measurement units to indicate the density of aerosols in a sample of air: Particles/meter^3 or micrograms/meter^3 (μg/m^3).

The average aerosol density for Los Angeles, California between October 2002 and September 2003 was 40 μg/m^3. 50% of these aerosols by mass were particulates with diameters of about 5 microns, while the remaining aerosols were mostly 500 nanometers in size.

Problem 1 – What was the density of the aerosols in particles/m^3 in each case, if the aerosols were small solid spheres with a density of 3.0 gm/cm^3?

Problem 1 –What was the density of the aerosols in particles/m^3 in each case, if the aerosols were small solid spheres with a density of 3.0 gm/cm^3?

Answer: Aerosol masses:

<u>5 micron case</u>: R = 2.5×10^{-4} cm then V = 4/3 π $(2.5\times10^{-4}\text{cm})^3$ = 6.5×10^{-11} cm^3.
Mass = 3.0 gm/cm^3 x 6.5×10^{-11} cm^3 = 2.0×10^{-10} gm.

<u>500 nanometer case</u>: R = 5.0×10^{-5} cm. V = 4/3 π $(5.0\times10^{-5}\text{ cm})^3$ = 5.2×10^{-13} cm^3.
Mass = 3.0 gm/cm^3 x 5.2×10^{-13} cm^3 = 1.6×10^{-12} gm.

Since 50% of the aerosols were in each category and the total density was 40 μg/m^3,

<u>5-micron case</u>: $$\text{number} = 20\ \mu g/m^3 \times \frac{1\text{ gm}}{10^6\ \mu g} \times \frac{1\text{ particle}}{2.0\times10^{-10}\text{ gm}} = \mathbf{10^5\ particles/m^3}$$

<u>500 nm case</u>: $$\text{number} = 20\ \mu g/m^3 \times \frac{1\text{ gm}}{10^6\ \mu g} \times \frac{1\text{ particle}}{1.6\times10^{-12}\text{ gm}} = \mathbf{1.2\times10^6\ particles/m^3}$$

So although there were an equal amount of particles by their <u>total mass</u>, the small particles were 120 times more <u>numerous</u> as individual particles in the air samples.

AQI	$PM_{2.5}$ (μg/m^3)	PM_{10} (μg/m^3)	Air Quality Descriptor
0–50	0.0–15.4	0–54	Good
51–100	15.5–40.4	55–154	Moderate
101–150	40.5–65.4	155–254	Unhealthy for Sensitive Groups
151–200	65.5–150.4	255–354	Unhealthy
201–300	150.5–250.4	355–424	Very unhealthy

Because of their impacts to health, the US Environmental Protection Agency monitors the level of aerosols in the atmosphere (troposphere) for two categories: Large aerosols (PM_{10}) with diameters near 10 microns, and small aerosols ($PM_{2.5}$) with diameters near 2.5 microns (μm). The Air Quality Index (AQI) relates the density of each aerosol type (measured in micrograms per cubic meter or μg/m^3) to health risk as shown in the table above.

Problem 1 - Suppose the two types of aerosol particles have a density of 2000 kg/m^3. Assuming that each particle is a perfect sphere, what are the average masses of each type of aerosol particle in kilograms?

Problem 2 – Based on your estimate of the aerosol particle masses in Problem 1, how many aerosol particles of each type would be present in a 1 cubic meter volume of air of the AQI was 150?

Problem 1 - Suppose the two types of aerosol particles have a density of 2000 kg/m^3. Assuming that each particle is a perfect sphere, what are the average masses of each type of aerosol particle in kilograms?

Answer: Volume = $4/3 \pi R^3$,

$PM_{2.5}$ aerosols: For R = 1.3 microns, R = 1.3×10^{-6} meters so
V = $1.333 \times 3.141 \times (1.3 \times 10^{-6}\ m)^3$
= $9.2 \times 10^{-18}\ m^3$.

Mass = density x volume, so

M = $2000 \times 9.2 \times 10^{-18}$
= **1.8×10^{-14} kilograms.**

PM_{10} aerosols: R = 5 microns so

V = $1.333 \times 3.141 \times (5.0 \times 10^{-6}\ m)^3$
= $5.2 \times 10^{-16}\ m^3$, then

Mass = $2000 \times 5.2 \times 10^{-16}$
= **1.0×10^{-12} kilograms.**

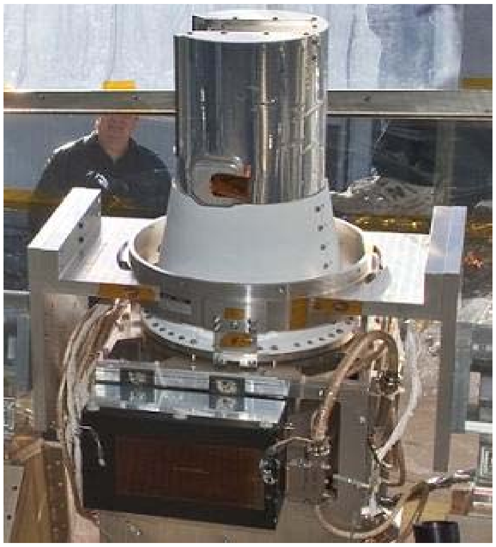
Problem 2 – Based on your estimate of the aerosol particle masses in Problem 1, how many aerosol particles of each type would be present in a 1 cubic meter volume of air of the AQI was 150?

Answer: The table indicates that for an AQI of 150, the density of the PM_{10} particles would be 254 $\mu g/m^3$. Since the mass of such an aerosol particle is about 1.0×10^{-12} kilograms, we have

N = $2.54 \times 10^{-6}\ \mu g/m^3 \times (1\ kg/1000\ gm) \times (1\ particle/1.0 \times 10^{-12}\ kg)$
= **2500 particles/meter^3.**

The table indicates that for an AQI of 150, the density of the $PM_{2.5}$ particles would be 65.4 $\mu g/m^3$. For $PM_{2.5}$ aerosols the density is 65.4 mg/m^3. The average mass is 1.8×10^{-14} kg, so

N = $65.4 \times 10^{-6}\ \mu g/m^3 \times (1\ kg/1000\ gm) \times (1\ particle/1.8 \times 10^{-14}\ kg)$
= **3.6×10^6 particles/meter^3.**



The Stratospheric Aerosol and Gas Experiment III (SAGE III), the sensor will be installed on the International Space Station (ISS) sometime in 2014. SAGE III will be using the sun and moon as light sources to measure how well the ozone layer is recovering and replenishing itself.

The 76-kilogram instrument will be carried in a Dragon supply module, and launched on a SpaceX, Falcon 9 rocket from NASA's Kennedy Space Center. After installation on the ISS and a 30-day check out, it will start taking data at a rate of about 200 megabytes every day. SAGE-III has a pointing accuracy of 0.025 degrees and requires 700 kilowatt hours of electricity every year.

Problem 1 - A single DVD can store 4.4 gigabytes of data. How many DVDs of data will the SAGE-III experiment generate during its 3-year mission onboard the ISS if 1 gigabyte = 1000 megabytes?

Problem 2 - If 1 kilogram equals 2.2 pounds, what is the weight of the SAGE-III instrument?

Problem 3 - A single 100-watt light bulb that is turned on for 10 hours will consume 1000 watt-hours of electricity, which is called 1 kilowatt-hour. How many watts will the SAGE-III instrument use if it is turned on for one full year?

Problem 4 - The SAGE-III instrument can change its direction of pointing by as little as 0.025 degrees. This is the same angle as the width of a dime (1 cm in diameter) if it were viewed at a distance of 23 meters. If two people stood one meter apart, how far away would they have to be standing from you to subtend the same angle? (Hint: Use proportions)

Problem 1 - A single DVD can store 4.4 gigabytes of data. How many DVDs of data will the SAGE-III experiment generate during its 3-year mission onboard the ISS if 1 gigabyte = 1000 megabytes?

Answer: The instrument produces 200 Mby of data every day. In 3 years it will produce 3 years x (365 days/year) x (200 Mby/day) = 219,000 Mby. Then:

219,000 Mby x (1 Gby/1000 Mby) x (1 DVD/4.4 Gby) = 49.8 DVDS. In terms of the total needed, we have **50 DVDs**.

Problem 2 - If 1 kilogram equals 2.2 pounds, what is the weight of the SAGE-III instrument?

Answer: 76 kg x (2.2 pounds/1 kg) = 167.2 pounds or **167 pounds**.

Problem 3 - A single 100-watt light bulb that is turned on for 10 hours will consume 1000 watt-hours of electricity, which is called 1 kilowatt-hour. How many watts will the SAGE-III instrument use if it is turned on for one full year?

Answer: The SAGE-III instrument uses 700 kWh of electricity. Then:

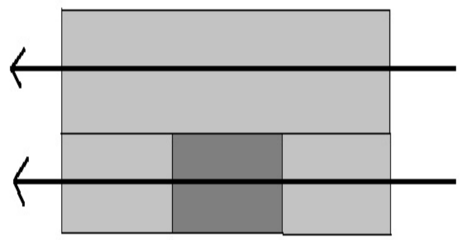
700 kWh x (1000 W/kW) x (1 year/ 365 days) x (1 day/ 24 hours) = **80 watts.**

Problem 4 - The SAGE-III instrument can change its direction of pointing by as little as 0.025 degrees. This is the same angle as the width of a dime (1 cm in diameter) if it were viewed at a distance of 23 meters. If two people stood one meter apart, how far away would they have to be standing from you to subtend the same angle?

Answer:

$$\frac{1\text{ cm}}{23\text{ meters}} = \frac{1\text{ meter}}{X}$$

convert all units to meters: $\frac{0.01\text{ m}}{23\text{ m}} = \frac{1\text{ m}}{X}$ then X = 23/0.01 = **2300 meters**



This figure shows two light rays from the Setting sun at two different altitudes above Earth. The top ray goes through a medium with fewer aerosols and a lower optical Depth.

The bottom ray passes closer to Earth's surface where a portion of the atmosphere is denser and has a different concentration of aerosols, making the dark region a higher optical depth.

The bottom atmosphere 'slab' is divided Into three equal sections, each 3,000 kilometers in length. The total width of the atmosphere being modeled in the example is 9,000 kilometers wide.

Opacity is a term used to describe the passage of light through a material, and is defined by the basic exponential formula

$$I = I_0 e^{-\tau}$$

Opaque materials have large positive values for τ, while transparent (translucent) materials have very low values for τ.

The quantity τ is also called the optical depth of a medium, and is proportional to both the length of the path taken by light through the medium, and the density of the absorbing particles. We can write this as

$$\tau = C\,N\,x$$

where x is in units of meters,
N is in units of particles/meter^3, and
C is a constant that is different for each kind of aerosol and has the units m^2/particle.

The Sage-III mission is designed to determine the product CN by looking at the extinction of sunlight along many different paths through the atmosphere given by x as shown in the figure to the left.

Problem 1 – Suppose that in the figure shown above, the SAGE-III instrument measures an extinction of light by 0.9991 in the top layer and 0.995 in the bottom layer. What are the total optical depths of each layer?

Problem 2 - From the information given in the figure, write two equations that relate the total optical depth to the contributions from each aerosol component and solve them to find the product CN for each aerosol.

Problem 3 – Suppose that the constants, C , for each aerosol type are known from models of Earth's atmosphere and that they are $C_A = 1.34\times10^{-17}$ km^2 /particle and $C_B = 7.5\times10^{-18}$ km^2 /particle, what are the densities of the two aerosols in A) particles/kilometer^3? B) particles/meter^3?

Problem 1 – Suppose that in the figure shown above, the SAGE-III instrument measures an extinction of light by 0.9994 in the top layer and 0.995 in the bottom layer. What are the total optical depths of each layer?

Answer: $I=I_0 e^{-\tau}$ and

Top layer: $I/I_0 = 0.9994$ so $\ln(0.9994) = -\tau$ and so $\tau =$ **0.0006**

Bottom layer: $I/I_0 = 0.995$ so $\ln(0.995) = -\tau$ and so $\tau =$ **0.005**

Problem 2 - From the information given in the figure, write two equations that relate the total optical depth to the contributions from each aerosol component and solve them to find the product CN for each aerosol.

Answer:

Top Layer aerosol: $0.0006 = CN_A \times (9000\text{ km})$

so $CN_A = 0.0006/9000\text{km}$

$= 6.7\times10^{-8}\text{ km}^{-1}$

Bottom Layer aerosol:

$0.005 = CN_A (3,000\text{ km}) + CN_B (3,000\text{ km}) + CN_A (3,000\text{ km})$

$0.005 = 2 (6.7\text{x}10\text{-}8\text{ km-}1)(3,000\text{ km}) + CN_B (3,000\text{ km})$

$0.005 = 0.0004 + CN_B (3,000\text{ km})$

$0.0046 = CN_B (3,000\text{ km})$

So $CN_B = 0.0046/3000\text{km}$

$= 1.5\times10^{-6}\text{ km}^{-1}$

So, **$CN_A = 6.7\times10^{-7}\text{ km}^{-1}$ and $CN_B = 1.5\times10^{-6}\text{ km}^{-1}$**

Problem 3 – Suppose that the constants, C , for each aerosol type are known from models of Earth's atmosphere and that they are $C_A = 1.34\times10^{-17}\text{ km}^2$ /particle and $C_B = 7.5\times10^{-18}\text{ km}^2$/particle, what are the densities of the two aerosols in A) particles/kilometer^3? And B) particles/meter^3?

Answer: Aerosol A: $(1.34\times10^{-17}\text{ km}^2/\text{particle})\ N_A = 6.7\times10^{-7}\text{ km}^{-1}$

so **$N_A = 5.0\times10^{10}$ particles/km^3**

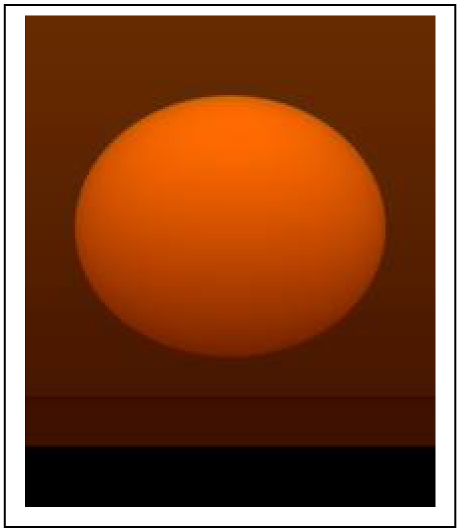
Aerosol B: $(7.5\times10^{-18}\text{ km}^2/\text{particle})\ N_B = 1.5\times10^{-6}\text{ km}^{-1}$

so $N_B =$ **2.0×10^{11} particles/km^3**

Aerosol A: 5.0×10^{10} particles/km^3 x (1km/1000meters)^3 = **50 particles/meter^3**

Aerosol B: 2.0×10^{11} particles/km^3 x (1 km/1000 meters)^3 = **200 particles/meter^3**

Three Mathematical Ways to Describe Light Extinction



When light passes through a medium it can lose some of its intensity. Scientists call this extinction. Depending on what properties they want to highlight in a calculation or a measurement, different ways of expressing extinction by a medium have arisen.

Opacity - Symbol τ : $I = I_0 e^{-\tau}$

Decibels - Symbol D : $I = I_0 10^{-D/10}$

Extinction Coefficient - Symbol C : $I = I_0 e^{-Cx}$

Problem 1 - If $e = 10^{0.434}$, and $10 = e^{2.3}$ write all three equations A) in base-10 B) in base-e.

Problem 2 – In base-10, what is the relationship between τ, D and C?

Problem 3 – In base-e, what is the relationship between τ, D and C?

Problem 4 - The SAGE-III instrument measures a 1 Decibel (1 dB) drop in the sun's brightness along a path through the atmosphere of x=2000 km. What is the optical depth and extinction coefficient for this region of the atmosphere?

Problem 1 - If $e = 10^{0.434}$ and $10 = e^{2.3}$ write all three equations A) in base-10 B) in base-e.

A) $I = I_0 e^{-\tau}$ $I = I0 (10^{0.434})^{\tau}$ so $\mathbf{I = I_0 10^{-0.434\tau}}$

$I = I_0 10^{-D/10}$ unchanged so $\mathbf{I = I_0 10^{-D/10}}$

$I = I_0 e^{-Cx}$ $I = I_0 (10^{0.434})^{(-Cx)}$ so $\mathbf{I = I_0 10^{-0.434Cx}}$

B) $\mathbf{I = I_0 e^{-\tau}}$ unchanged

$I = I_0 (e^{2.3})^{-D/10}$ so $\mathbf{I = I_0 e^{-0.23D}}$

$\mathbf{I = I_0 e^{-Cx}}$ unchanged

Problem 2 – In base-10, what is the relationship between τ, D and C?

Answer: Just set the exponential factors equal to each other in Problem 1 A:

$-0.434\tau = -D/10 = -0.434Cx$ so after simplifying we get $\mathbf{\tau = 0.23D = Cx}$

Problem 3 – In base-e, what is the relationship between τ, D and C?

Answer: Set the exponential factors equal to each other in Problem 1 B: $\mathbf{\tau = 0.23D = Cx}$

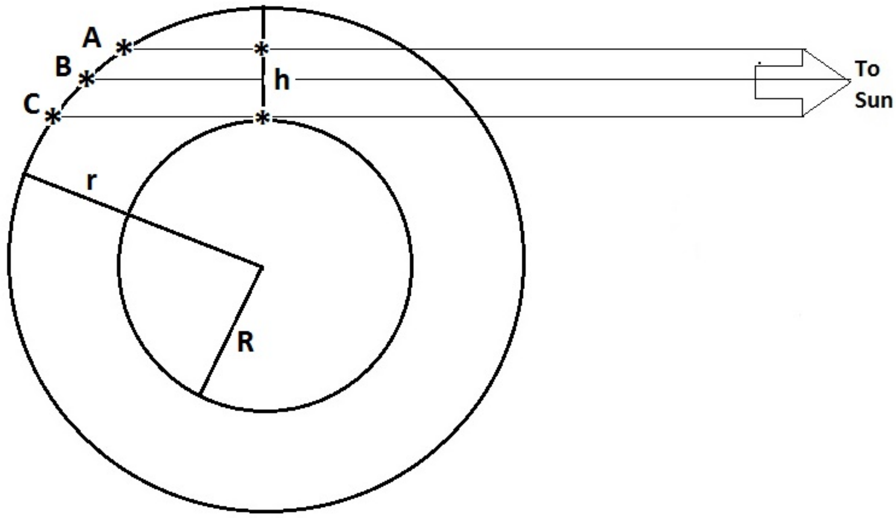
Problem 4 - The SAGE-III instrument measures a 1 Decibel (1 dB) drop in the sun's brightness along a path through the atmosphere of x=2000 km. What is the optical depth and extinction coefficient for this region of the atmosphere?

Answer: For 1 dB, and from Problem 2 (or 3!) we have

$\tau = 0.23$ D so
$\tau = 0.23 \times 1$ dB
$\tau =$ **0.23**.

For 1 dB and for x = 2000 km, we have
0.23 dB = Cx and so
0.23 = 2000C and so
C = 0.000115 km^{-1}.

SAGE-III Sunset and Sunrise Geometry



The SAGE-III instrument on the International Space Station orbits Earth at a distance of r = 6,730 km from the center of Earth. The radius of Earth is R = 6,378 km. The time for one complete orbit is about 90 minutes. As it travels from Point A to C in the figure, the height of the sun, h, above the edge of Earth decreases to zero and astronauts observe a sunset. Each time SAGE-III observes a sunrise or sunset, its instruments measure the brightness of the sun. From this sun-dimming information scientists can determine the aerosol content of the stratosphere above an altitude of 10 km.

Problem 1 - Use the Pythagorean Theorem to determine the length of a chord for a given value of h for h < 100 km.

Problem 2 - Most of the sunlight extinction will happen within a height of h = 40 km. About how long is the total length of the chord near the sunset point in the orbit?

Problem 3 – About how many seconds will it take for the sunset to progress from h=40km to h=0 km?

Problem 1 - Use the Pythagorean Theorem to determine the length of a chord for a given value of h for h < 100 km.

Answer: $l^2 = r^2 - (R+h)^2$ so

Length = $2l = 2\,(r^2 - R^2 - 2Rh - h^2)^{1/2}$

Since R = 6378 and r = 6730 we have by simplifying

$L = 2\,(6730^2 - 6378^2 - 2(6378)h - h^2)^{1/2}$

Factor out 6730^2

Then $L = 2\,(6730)(1 - 0.90 - 0.00028h - (h/6730)^2)^{1/2}$

But h/6730 is never more than 100/6730 = 0.015 so we can ignore the h^2 term entirely!

So, $\mathbf{L = 13460(0.10 - 0.00028h)^{1/2}}$

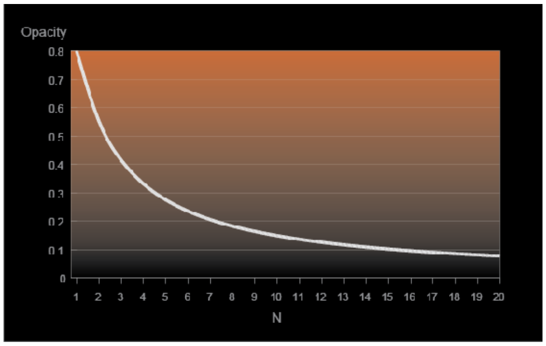
Problem 2 - Most of the sunlight extinction will happen within a height of h = 40 km. About how long is the total length of the chord near the sunset point in the orbit?

Answer: h = 0 at sunset so $L = 13460\,(0.10)^{1/2}$ = **4256 km**.

Problem 3 – About how many seconds will it take for the sunset to progress from h=40km to h=0 km?

Answer: The ISS will travel about 40 km in its orbit. Since the circumference of the circular orbit is $C = 2\,\pi\,(6730\text{ km}) = 42280$ km, and this takes 90 minutes, the sunset range of 40 km will be traversed in

$$\frac{40\text{ km}}{42280\text{ km}} \times 90\text{ minutes} \times (60\text{ sec/1 minute}) = \mathbf{5\ seconds}.$$



This curve shows how much light is transmitted through a stack of 20 filters if each filter reduces the transmitted light by 20%.

After the first filter, N=1, the light intensity has been reduced from 100% to 80%.

After passing through the second filter, N=2, the light intensity is reduced to
$100\% \times 0.8 \times 0.8 = 100\% \times (0.8)^2 = 64\%$.

The reduction of light brightness as it passes through an aerosol cloud is not an additive process, but a multiplicative one. The figure to the left shows how light brightness changes as it passes through stacks of filters of varying lengths from 1 to 20. Note that the curve is not a straight line as it would be if the dimming were additive.

Scientists measure the dimming of light by the distance at which the light brightness is reduced by exactly 2.718 times or from 100% to 36.8%. Because aerosols are very dilute, the distance to which the light intensity falls to 36.8% is typically measured in kilometers. A basic mathematical function that describes light attenuation is given by

$$I(x) = 1.0\ e^{-x/L}$$

where L is the attenuation distance in kilometers and X is the length of the path through the aerosols.

Problem 1 – Suppose that for a particular cloud of aerosols the attenuation distance is 2 kilometers and the actual thickness of the cloud is 0.5 kilometers. What will be the light intensity to the nearest percent for a light ray passing through this cloud?

Problem 2 – For convenience, the attenuation distance L is usually reported as the extinction coefficient C = 1/L in units of km^{-1}. A) What is the equation for I(x) in terms of C? B) What is the value for I(x) in percent for a cloud with C = 0.20 km^{-1} and a cloud thickness of x=20 km?

Problem 3 – The SAGE-III instrument will measure the stratospheric aerosols, which have an average extinction of 1.2×10^{-4} km^{-1}. Light from the rising and setting sun will be measured along a path through the stratosphere that is about 3,000 km in length. What is the intensity of sunlight reaching the SAGE-III instrument?

Problem 1 – Suppose that for a particular cloud of aerosols the attenuation distance is 2 kilometers and the actual thickness of the cloud is 0.5 kilometers. What will be the light intensity to the nearest percent for a light ray passing through this cloud?

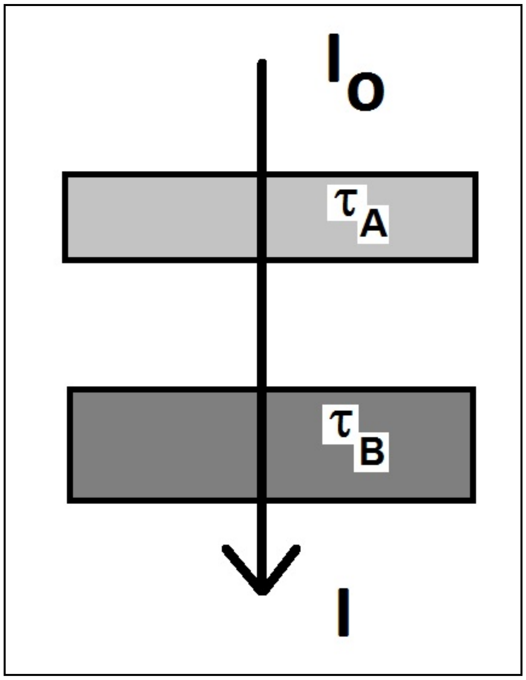
Answer: L = 2 km and x = 0.5 km so I = 100% x e(-0.5/2) = **78%**

Problem 2 – For convenience, the attenuation distance L is usually reported as the extinction coefficient C = 1/L in units of km^{-1}. A) What is the equation for I(x) in terms of C? B) What is the value for I(x) in percent for a cloud with C = 0.20 km^{-1} and a cloud thickness of x=20 km?

Answer: A) **I(x) = 1.0 e^{-Cx}** B) I(20) = 100% $e^{(-0.2x20)}$ = **1.8%**

Problem 3 – The SAGE-III instrument will measure the stratospheric aerosols, which have an average extinction of $1.2x10^{-4}$ km^{-1}. Light from the rising and setting sun will be measured along a path through the stratosphere that is about 3,000 km in length. What is the intensity of sunlight reaching the SAGE-III instrument?

Answer: I(3000km) = 100% $e^{(-0.00012)(3000)}$ = **69.7%**.



Opacity is a term used to describe the difficulty for light to travel through a medium. A rain cloud with high opacity appears almost black as it hangs in the sky above your head. On the other hand, frosted glass lets some light through and has low opacity.

Extinction is a term that describes how much the intensity of light has been reduced as it passes through a medium.

The terms opacity and extinction are often used interchangeably, but mathematically, scientists who work with light define them differently. One basic equation that relates them together is:

$$I = I_0 \, e^{-\tau}$$

I_0 is the initial intensity of the light as it strikes the front surface of the medium. **I** is the intensity of the light after it has left the medium, and τ is the opacity of the medium that the light has passed through. A high opacity (opaque) medium is one for which τ is large, and this causes the light leaving the medium, I, to be reduced in intensity, which we call extinction.

When light passes through two different materials, one after the other, the final intensity is just

$$I = (I_0 \; e^{-A})e^{-B} \quad \text{or} \quad I = I_0 \, e^{-(A+B)}$$

where A is the opacity (τ) of the first medium and B is the opacity (τ) of the second medium.

Problem 1 – Show that for three different mediums, the total opacity of the materials is given by the formula

$$\tau = \tau_A + \tau_B + \tau_C$$

Problem 2 - A photographer is given three different filters with opacities of τ_A= 5.2, τ_B = 1.3 and τ_C = 0.5. He thinks that by placing the most opaque filter last that the light will be slightly brighter when it enters the camera. Do you think that this will work?

Problem 1 – Show that for three different mediums, the total opacity of the materials is given by the formula

$$\tau = \tau_a + \tau_b + \tau_c$$

Answer: From our example, and extended for three medii:

$$I = ((I0\ e^{-A})e^{-B})e^{-C})\ \text{ or }\ I = I0\ e^{-(A+B+C)}$$

But the total opacity is just

$$I = I_0 e^{-\tau},$$

so since $A = \tau_A$, $B=\tau_B$ and $C=\tau_C$ we have

$$I0e^{-\tau} = I0e^{-(\tau_A + \tau_B + \tau_C)}$$

And so

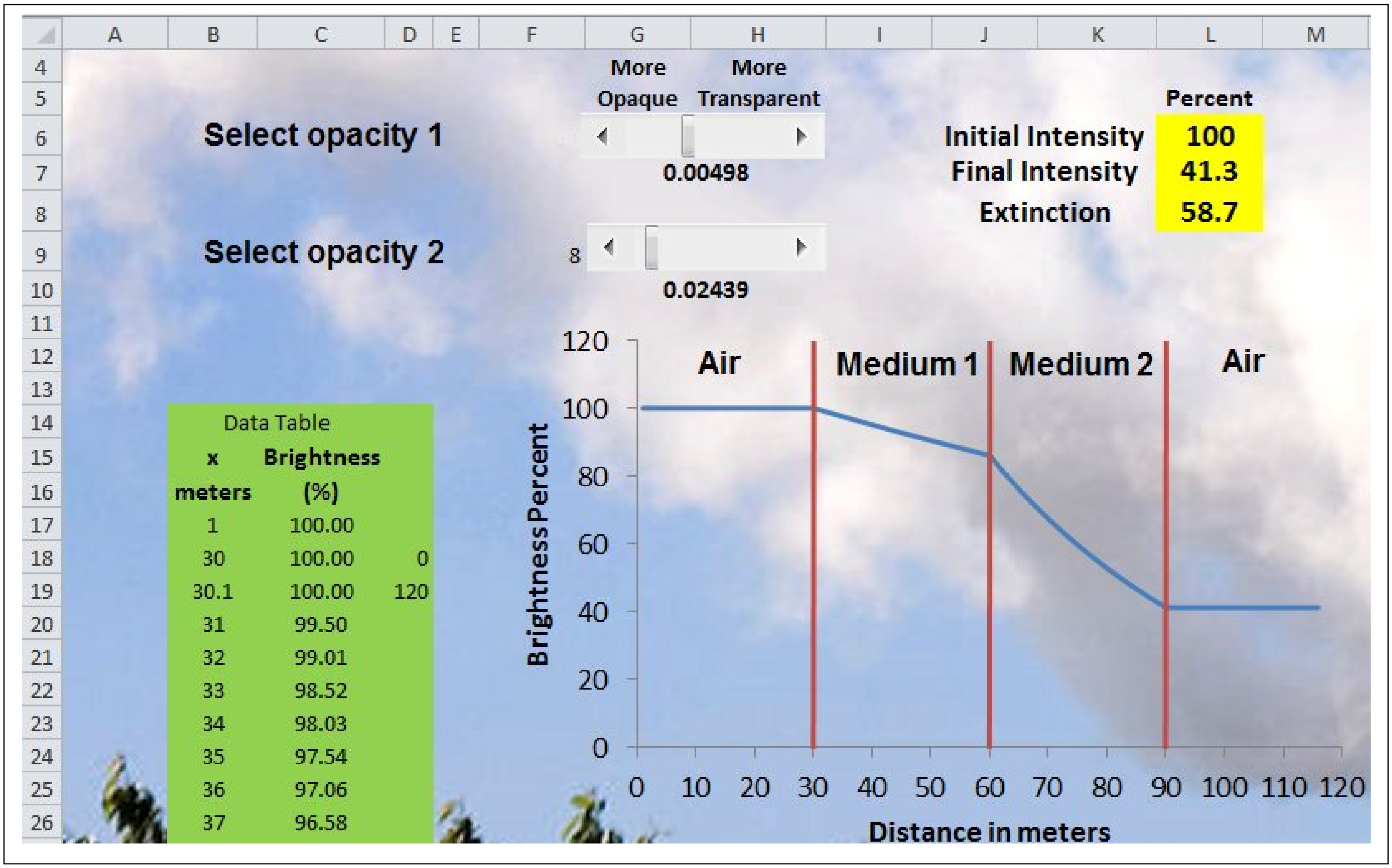
$$\tau = \tau_A + \tau_B + \tau_C$$

Problem 2 - A photographer is given three different filters with opacities of τ_A= 5.2, τ_B = 1.3 and τ_C = 0.5. He thinks that by placing the most opaque filter last that the light will be slightly brighter when it enters the camera. Do you think that this will work?

Answer: This will not work because the final opacity is the sum of the opacities of the three filters, and the order in which you add the filters does not matter in the sum. You will still get a final opacity of 5.2+1.3+0.5 = 7.0 and a drop in brightness by a factor of

$$I = I_0\ e^{-7} \quad \text{or}\ \ 0.00091$$

Exploring Opacity



This lab allows you to examine how the change in the opacity of a material affects its ability to transmit light. The basic formula that relates opacity to transmission is given by

$$I = I_o \, e^{-\tau}$$

Where **I** is the intensity of the light after passing through the absorber,

I_o is the initial intensity of the light, and

τ is the optical depth of the medium.

Optical depth is a product of two factors: the opacity of the medium, A, and the distance that light has traveled through the medium, X. Opacity has the units of 1/length or in SI units, meter^{-1}. The product A X has no units, and represents the optical depth, τ, in the formula.

The program is set up with two materials back to back. This would be like the photographer using two filters attached to the lens of her camera.

Here is how the program works:

There are two sliders you can use to adjust the opacity, A, of each filter. The slider for Medium 1 is located at G6 – H6, and the slider for Medium 2 is located at G9-H9. The value for the opacity you select by moving the sliders is given in cell G7 for Medium 1 and G10 for Medium 2.

The experiment is scaled so that the absorbing materials are each 30 meters wide, but similar results will be obtained if you rescaled the experiment to 3 centimeters or 3 millimeters.

The percentages in column L give the initial intensity of the light falling on Medium 1 at a distance of 30 meters (cell L6), The final intensity of the light as it emerges from Medium 2 at a distance of 90 meters (cell L7) and the difference between these two percentages to indicate how much of the light has been absorbed (cell L8).

In the data table, Column B gives the distance in meters through the air and the two materials starting from the left-hand edge at ‘0 meters’. In Column C the remaining intensity is calculated. For travel through the air there is no loss of brightness. It is assumed that I(30 meters) = 100%. This is the value entered into cell L6. Once you are inside one of the materials, the loss of light is given by the formula

$$\mathbf{colC = I(30\ meters)\ e^{(-G7(colB-30))}}$$

for the first medium which has the slider-selected opacity given in cell G7. This opacity value is multiplied by the distance inside the medium from the incident edge at x=30 meters to get the optical depth τ = G7(colB-30). This formula is continued down the column until we reach the boundary of the second medium at x=60 meters where the formula then changes to

$$\mathbf{colC = I(60\ meters)\ e^{(-G10(colB-60))}}$$

where

$$\mathbf{I(60\ meters) = I(30\ meters)\ e^{(-G7(60-30))}}$$

This formula is then continued until we reach the right-hand air boundary at 90-meters where the intensity becomes

$$\mathbf{I(90) = I(60)\ e^{(-G10(90-60))}}$$

and remains constant thereafter. This is the value entered into cell L7.

Exploration Problem 1 - Some photographers think that by placing the denser filter followed by the less-dense filter, that you will get more light absorbed than if you placed the less-dense filter first. Does this idea make sense?

Exploration Problem 2 – For what opacities will the amount of extinction from Medium 1 and 2 add in a linear fashion (i.e A+B=C) For what range of opacities will this linear relationship fail?

Exploration Problem 3 – An engineer needs to build an instrument to filter an exact amount of light. He already has one filter that has an opacity of 0.009 $meters^{-1}$, so he needs to determine what the opacity of Medium 2 needs to be so that the incoming light is reduced by exactly 50%. What is the opacity of Medium 2 that will do the trick?

Exploring Sunlight Extinction

As light passes through a medium, some of it is reflected, some of it is scattered, and some of it is absorbed by the medium. For example, raindrops in a cloud reflect light to make the cloud appear white, but some of the light also penetrates the cloud. Some of this light experiences multiple reflections within the cloud (called scattering) and may eventually find its way out of the cloud. Some of the light entering a cloud can also be absorbed by atoms and droplets in the cloud. The remaining light exits the cloud, but at reduced brightness because of all the losses along the way. The difference between the percentage of light entering a medium and exiting the medium is called the extinction. By studying the amount of extinction at various angles and locations, scientists can create a model of what kinds of particles (atoms or aerosols), and how many (their concentration), are producing the observed effect.

This mathematical model lets you explore how light intensity changes during a sunrise or sunset event as viewed from Earth orbit. Astronauts observe 16 sunrises and sunsets every day as the International Space Station orbits Earth every 90 minutes. Data gathered from the ISS by the Stratospheric Aerosol and Gas Experiment (SAGE III) during these sunrises, sunsets and similar events for the moon, will let scientists study the concentration and types of aerosols found in the stratosphere. This plays an important role in global warming, and will improve scientific models for climate change.

How the Program Works.

The program is designed to model Earth-sized planets with an atmosphere. First you have to establish the scale of the model by entering then radius of the planet and the orbit altitude from the surface. The screen-shot below shows choices that represent our Earth (cell B2 =

	Earth Surface		ISS Orbit		Sight Line	
theta	x	y	x	y	x	y
0.00	6378	0	6778	0	0	6778
0.02	6377	111	6777	118	100	6749
0.03	6374	221	6774	235	200	6720
0.05	6369	332	6769	352	300	6690
0.07	6363	442	6762	470	400	6661
0.09	6354	552	6753	587	500	6632
0.10	6344	662	6741	704	600	6603
0.12	6331	772	6728	821	700	6574
0.14	6317	882	6713	937	800	6544
0.16	6301	991	6696	1053	900	6515

6378 km) and the orbit of the ISS (cell B4 = 400 km). After entering these values, the spreadsheet will immediately update the entries in columns B, C, D and E which are used to plot the X and Y coordinates for the two circles shown in the figure. The image of Earth is shown at a scale that matches choices for B2 and B4, but can also be used to compare Earth to larger planet choices. The coordinates for the two circles are obtained from the polar equations for a circle: $X = R \cos(\theta)$ and $Y = R \sin(\theta)$. The value for q in radians is selected from column A from 0 to π, and only a half-circle is plotted.

The next calculation is for the sightline to the sun, represented by the purple line in the figure. This is a simple linear equation that starts at a y-intercept of y0= (B2+B4) with x-values X = column F, and the equation is Y = y0 + M(column F).

The height of the line above the Earth at its closest point is given by

J6 = (B2+B4) sin(J5) –B2,
where J5 = -atan(1/J4) in radians
and J4 = -0.004*K4.

The value for K4 is selected by the slider from values between 1 and 100. As the slider values change, the sightline varies from being tangent to the orbit of the ISS (K4=0) to tangent to Earth surface (near K4=90). Values for K4 greater than 90 lead to unphysical answers in the model. The sun angle relative to the orbit is given in cell J7 and is just the angle in cell J5 converted into degree measure by J7 = 180*J5/3.1416.

Once the orientation of the sightline has been selected from the slider, the program calculates the amount of light absorption along the path to the ISS. The details of how this is done are displayed in the second section of the spreadsheet shown in the screen shot below.

Distance	Height	Density	Opacity
X	y		
0	400	0.0065	0.000001
100	373	0.0161	0.000002
200	347	0.0383	0.000006
300	322	0.0869	0.000013
400	299	0.1882	0.000028
500	277	0.3885	0.000058
600	257	0.7647	0.000115

Scale Height	30	km
Extinction Coeff.	0.0000015	km^-1
Step size	100	km
Total Opacity =	0.09	
Solar light Dimming=	9	percent

Once the linear equation for the sightline has been selected, we can use the linear equation to determine the height of each point along the line from the surface of Earth using the formula:

$$J = \sqrt{x^2 + R^2 - 2xR\cos(J5)} - B2$$

where x = col I44 to col I86,
and R = B2+B4.

To determine the density of the aerosols at this height, we use a simple ‘exponential atmosphere’ model D = 4000 $e^{(-J/P42)}$ where cell P42 is the value you select for the scale height of the atmosphere in kilometers. Once this selection is made, the values for the density along the sightline in column K44 to K86 are computed.

Finally, we use a simple model for the relationship between aerosol density and opacity using the formula

Opacity = density x extinction coefficient x step size,
given by the product colJ = colK x P43 x P44,

where the user selects the extinction coefficient in cell P43 and the step size in kilometers in cell P44. This step size is also used to increment the values for X in column I.

The total optical depth along the sightline given in cell P45 is just the sum of the values in column L. The amount of light extinction is given in cell P47 based on the formula

$$P47 = 100\%\ e^{(-P45)}$$

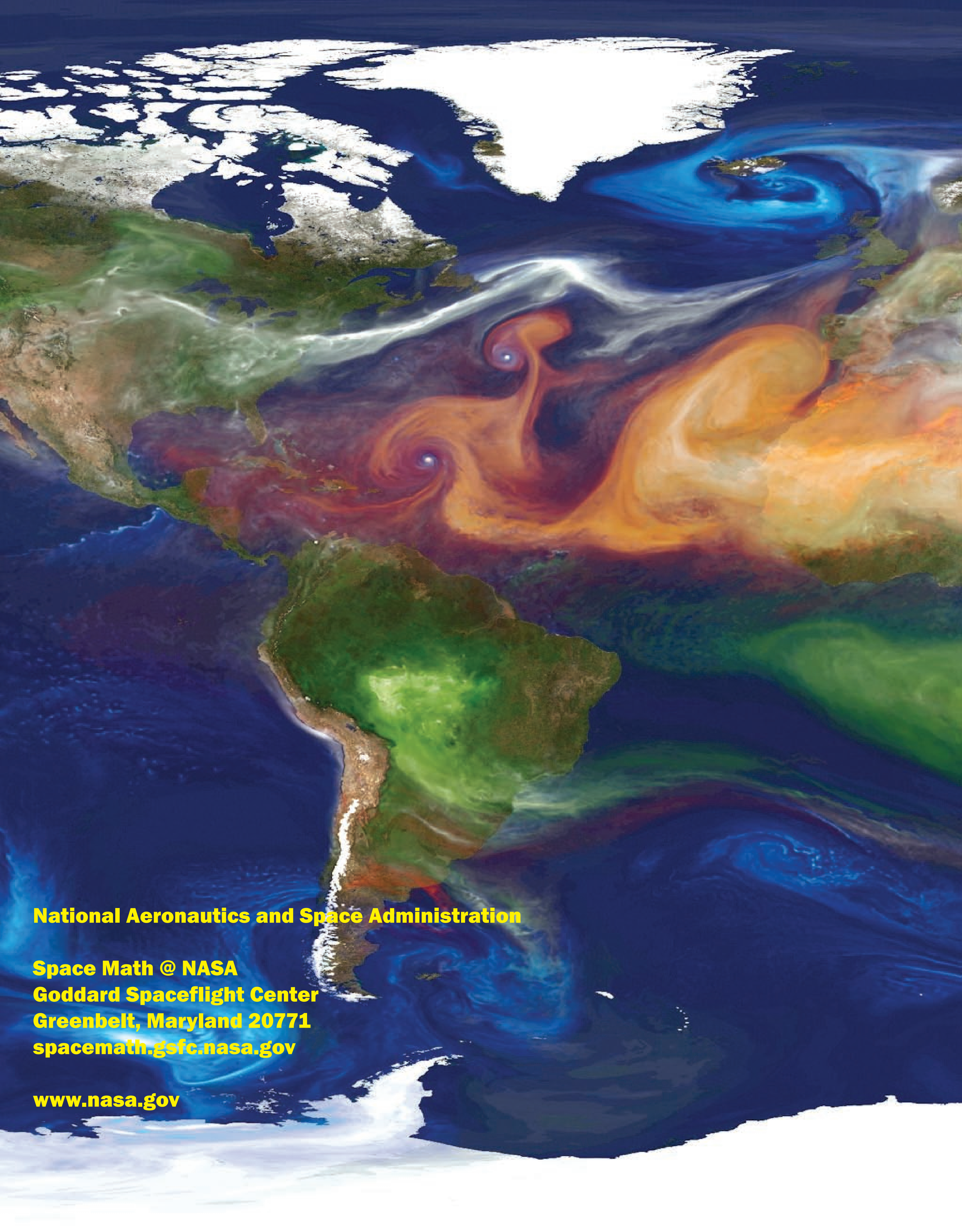
100% means that all of the incident sunlight is extinguished by the time it arrives at the ISS.

Exploration Problem 1 – For what values of the model does it give unphysical answers?

Exploration Problem 2 – What are the assumptions made in creating the model? How can you improve this model?

Exploration Problem 3 – At what point along the sightline is the opacity the greatest, and why is this?

Exploration Problem 4 – At the orbit of the ISS the direction to the sun changes by about 4 degrees per minute. How long will the sunrise extinction measurements made by the SAGE-III instrument last before the sunlight extinction has reached 10%?



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Space Math @ NASA
Goddard Spaceflight Center
Greenbelt, Maryland 20771
spacemath.gsfc.nasa.gov

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