

When we view a source of light at different distances, the light gets fainter as the distance increases. The figure to the left shows why this happens. At a distance of '2d', the same amount of light energy passes through 4 times the area that it did at the closer distance, so its brightness will 4 times fainter. This is called the **Inverse-Square Law**. Can you explain why it has this name?

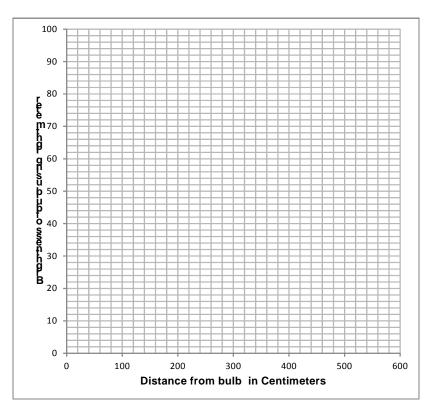
Our robotic light meter can be used to test the Inverse-Square law for an unknown light source by comparing its distance from the source with the light intensity its meter registers. Here's how we do this.

Step 1 – In a darkened room, turn on a bare 40-watt light bulb on the floor. Position the rover so that the light meter just registers '99' before changing to '98'. Measure this distance from the sensor to the bulb in centimeters. Enter the reading and the distance into a data table.

Step 2 – Move the rover directly away from the bulb until the light meter reads '50' and measure this distance. Record these numbers in your data table.

Step 3 - Repeat Step 2 for light readings of '25', '12', and ''6'.

Step 4 - Graph your data below and connect the dots with a smooth curve.



Problem 1– What would you predict as the distance where the light reading is '1'?

Problem 2 – What would happen if you used a 80-watt bulb instead of the 40-watt bulb?

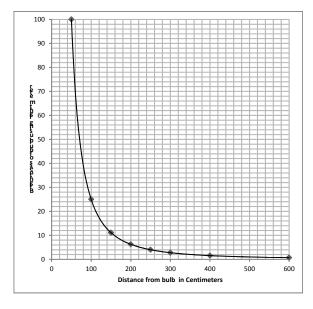
Problem 3 – Write a mathematical model for the brightness, B, of a light source, the distance, D, and the wattage, W, of the light source, if for 40 watts you get B=100 at a distance of 50 cm. What do you predict the brightness will be at 800 cm for a 250 watt bulb?

Answer Key

This is called the Inverse-Square Law. Can you explain why it has this name?

Answer: The same number of light rays go through 4 times the area A at twice the distance, 9 times the area of A at three times the distance, and since $4 = 2^2$ and $9 = 3^2$ the brightness decreases by $1/2^2 = \frac{1}{4}$ at twice the distance and $1/3^2 = \frac{1}{9}$ at three times the distance, which follows a $1/d^2$ or Inverse-Square model.

Let's suppose that the light meter data follows the curve graphed below, where we measured '99' at a distance of 50 cm:



Problem 1– What would you predict as the distance where the light reading is '1'?

Answer: Using the hypothetical graph above, we have to reduce the light intensity by 1/100 of its original level at 50 cm. Because $1/100 = 1/10^2$, we need to move to 10 times the distance or 10 x 50 cm = 500 cm.

Problem 2 – What would happen if you used a 80-watt bulb instead of the 40-watt bulb?

Answer: You can test what happens using a brighter bulb. Because there is twice as much light at every distance, the curve drawn above **moves upwards** by a factor of 80/40 = 2 times, so at a distance of 100 cm the light meter would read 25x2 = 50.

Problem 3 – Write a mathematical model for the brightness, B, of a light source, the distance, D, and the wattage, W, of the light source if for 40 watts you get B=100 at a distance of 50 cm. What do you predict the brightness will be at 800 cm for a 250 watt bulb?

Answer: $B = 100 \times \frac{(W/40)}{(D/50)^2}$ and so $B = 100 \times (250/40) / (800/50)^2 = 2.4$ units

Space Math