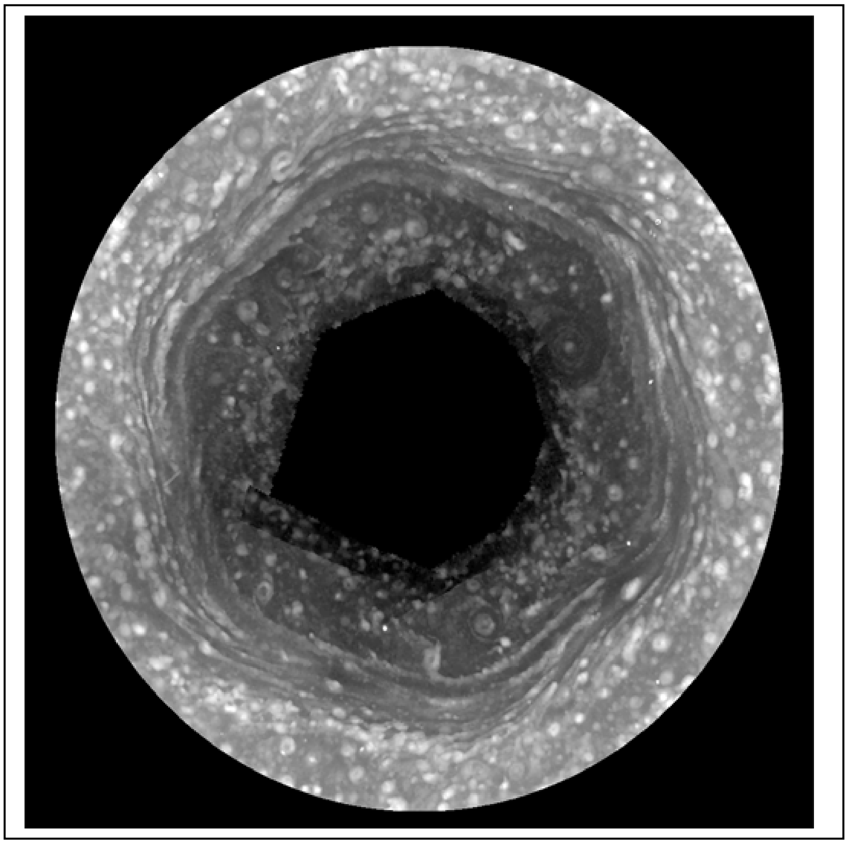
The Mysterious Hexagon on Saturn!



The Cassini spacecraft recently took this high-resolution image of Saturn's north-polar region, which was observed by the Voyager 1 and 2 spacecraft in the early-1980s to have a remarkable hexagonal jet stream! The white spots are individual clouds, much like fair-weather cumulus clouds on Earth. The interior distance between opposite vertices of the hexagon is 25,000 kilometers, and the estimated speed of the winds along the walls of the hexagon is about 100 meters/sec. Use a millimeter ruler, or your knowledge of hexagons, to answer these questions:

Problem 1 - How long does it take the jet stream to make one complete circuit of the hexagon; A) In hours?, B) In days?

Problem 2 - The Earth has a radius of 6,378 kilometers, from a scaled drawing of the hexagon and Earth, how many Earths can fit inside the hexagonal area?

Problem 3 - Acceleration is a measure of the change in the speed and/or direction of motion per unit time interval. A) From the hexagon figure, how much time, T, elapsed in seconds as the velocity of the jet stream completely changed direction as it crossed a vertex region? B) If the total velocity change, V, passing across one vertex is about 173 meters/sec, what was the average acceleration of the jet stream defined as $a = V/T$ in meters/sec^2?

Problem 1 - How long does it take the jet stream to make one complete circuit of the hexagon; A) In hours?, B) In days?

Answer: The interior distance between opposite vertices is 67 mm, which corresponds to 25,000 kilometers, so the scale is 25,000 km/67 mm = 373 km/mm. The length of one side is 35 millimeters (or 13,000 km) so the full perimeter is 6 x 35 mm = 210 mm. Converting this to meters we get 210 mm x (373 km/mm) = 78,300 kilometers or 7.83×10^7 meters. The time taken is T = D/V so for D = 7.83×10^7 meters and V = 100 meters/sec we get T = 783,000 seconds. A) **217 hours** and B) **9 days**.

Problem 2 - The Earth has a radius of 6,378 kilometers, from a scaled drawing of the hexagon and Earth, how many Earths can fit inside the hexagonal area?
Answer: The earth circle will have a diameter of 2 x 6,378/373 = 34 millimeters. That will be about **2 Earth disks** spanning the inside of the hexagon. The area of a hexagon is given by $A = 3(3)^{1/2}/2\ L^2$ so the Saturn hexagon has an area of A = 3 (1.732)x(0.5) x $(13{,}000\ \text{km})^2$ = 440,000,000 km^2. The surface area of Earth is $A = 4\ \pi\ R^2$ = 510,000,000 km^2, so the hexagon has nearly the same surface area as all of earth.

Problem 3 - Acceleration is a measure of the change in the speed and/or direction of motion per unit time interval. A) From the hexagon figure, how much time, T, elapsed in seconds as the velocity of the jet stream completely changed direction as it crossed a vertex region? B) If the total velocity change, V, passing across one vertex is about 173 meters/sec, what was the average acceleration of the jet stream defined as a = V/T in meters/sec^2? Answer: A) The shape of the clouds near a vertex suggest that the turn a distance of about 8 millimeters in going from the direction along one face to the other .The physical distance is 8 mm x (373 km/mm) = 3,000 kilometers or 3 million meters. The speed is 100 meters/sec, so the time taken is 3,000,000 m/(100m/s) = **30,000 seconds**. B) The velocity change during this time was 173 meters/sec, so the acceleration A = V/T = (173 meters/sec) / (30,000 sec) = **0.006 meters/sec^2**.
Note, the acceleration of gravity at Earth's surface is 9.8 meters/sec^2, so the winds feel an acceleration of about 0.0006 Gs.

Note: From a study of vectors, the velocity difference between a flow along one face Vf, and the adjacent face is a vector 'turn' of 60 degrees, so the difference vector, Vd, has a magnitude of $Vd^2 = 2(Vf)^2 + 2(Vf)^2\cos(60)$ so for Vf = 100 m/sec, Vd = 173 meters/sec, which is the value used.