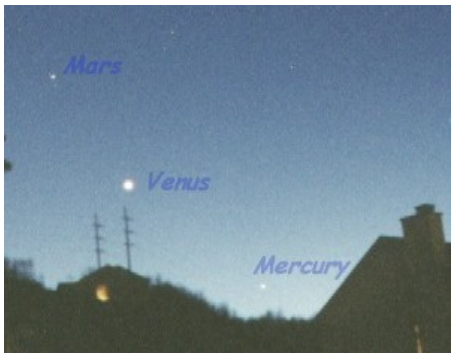
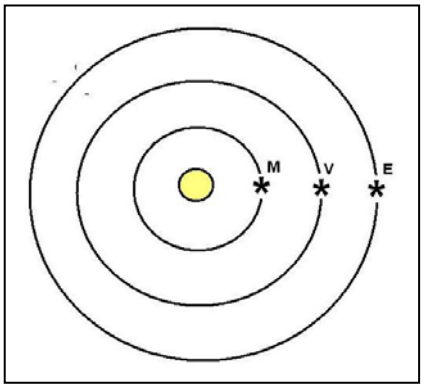
Planetary Alignments





One of the most interesting things to see in the night sky is two or more planets coming close together in the sky. Astronomers call this a conjunction. The picture to the left shows a conjunction involving Mercury, Venus and Mars on June 24, 2005.

As seen from their orbits, another kind of conjunction is called an 'alignment' which is shown in the figure to the lower left and involved Mercury, M, Venus, V, and Earth, E. As viewed from Earth's sky, Venus and Mercury would be very close to the Sun, and may even be seen as black disks 'transiting' the disk of the Sun at the same time, if this alignment were exact. How often do alignments happen?

Earth takes 365 days to travel one complete orbit, while Mercury takes 88 days and Venus takes 224 days, so the time between alignments will require each planet to make a whole number of orbits around the Sun and return to the pattern you see in the figure.

Suppose Mercury takes 1/4 earth-year and Venus takes 2/3 of an earth-year to make their complete orbits around the Sun. You can find the next line-up from two methods:

Method 1: Work out the three number series like this:
Earth = 0, 1, **2**, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12,…
Mercury = 0, 1/4, 2/4, 3/4 ,4/4, 5/4, 6/4, 7/4, **8/4**, 9/4, 10/4, 11/4, 12/4, 13/4, …
Venus = 0, 2/3, 4/3, **6/3**, 8/3, 10/3, 12/3, 14/3, 16/3, 18/3, 20/3, …
Notice that the first time they all coincide with the same number is at **2 years**. So Mercury has to go around the Sun 8 times, Venus 3 times and Earth 2 times for them to line up again in their orbits.

Method 2: We need to find the Least Common Multiple (LCM) of 1/4, 2/3 and 1. First render the periods in multiples of a common time unit of 1/12, then the sequences are:
Mercury = 0, 3, 6, 9, 12, 15, 18, 21, **24**,
Venus = 0, 8, 16, **24**, 32, 40, …
Earth, 0, 12, **24**, 36, 48, 60, …

The LCM is 24 which can be found from prime factorization:
Mercury: 3 = 3
Venus: 8 = 2 x 2 x 2
Earth: 12 = 2 x 2 x 3
The LCM the product of the highest powers of each prime number or 3 x 2 x 2 x 2 = 24. and so it will take 24/12 = **2 years.**

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Mercury = 7/30 x 365 = 85 days vs actual 88 days
Venus = 26/42 x 365 = 226 days vs actual 224 days
Earth = 1

The common denominator is 42 x 30 = 1,260 so the series periods are
Mercury = 7 x 42 = 294 so 7/30 = 294/1260
Venus = 26 x 30 = 780 so 26/42 = 780/1260
Earth = 1260 so 1 = 1260/1260

The prime factorizations of these three numbers are

294 = 2 x 2 x 3 x 7 x 7
780 = 2 x 2x 5 x 3 x 13
1260 = 2 x 2 x 3 x 3 x 5 x 7

LCM = 2 x 2 x 3 x 3 x 5 x 7 x 7 x 13 = 114,660

So the time will be 114,660 / 1260 = 91 years! In this time, Mercury will have made exactly 114,660/294 = 390 orbits and Venus will have made 114,660/780 = 147 orbits

Note to Teacher: Why did the example problem give only 2 years while this problem gave 91 years for the 'same' alignment? Because we used a more accurate approximation for the orbit periods of the three planets. Mercury actual period = 88 days but 1/4 earth-year = 91.25 days compared to 7/30 earth year = 85 days. Venus actual period = 224 days but 2/3 earth-year = 243 days and 26/42 earth-year = 226 days.

This means that after 2 years and exactly 8 orbits (8 x 91.25 = 730 days), Mercury will be at 8/4 x 365 = 730 days while the actual 88-day orbit will be at 88 x 8 = 704 days or a timing error of 26 days. Mercury still has to travel another 26 days in its orbit to reach the alignment position. For Venus, its predicted orbit period is 2/3 x 365 = 243.3 days so its 3 orbits in the two years would equal 3 x 243.3 days = 730 days, however its actual period is 224 days so in 3 orbits it accumulates 3 x 224 = 672 days and the difference is 730-672 = 58 days so it has to travel another 58 days to reach the alignment. In other words, the actual positions of Mercury and Venus in their orbits is far from the 'straight line' we were hoping to see after exactly 2 years, using the approximate periods of 1/4 and 2/3 earth-years!

With the more accurate period estimate of 7/30 earth-years (85 days) for Mercury and 26/42 earth-years (226 days) for Venus, after 91 years, Mercury will have orbited exactly 91 x 365 days/88 days = 377.44 times, and Venus will have orbited 91x365/224 = 148.28 times. This means that Mercury will be 0.44 x 88d = 38.7 days ahead of its predicted alignment location, and Venus will be 0.28 x 224 = 62.7 days behind its expected alignment location. Comparing the two predictions, Prediction 1: Mercury= - 26 days, Venus= - 58 days; Prediction 2: Mercury = +26 days and Venus = - 22 days. Our prediction for Venus has significantly improved while for Mercury our error has remained about the same in absolute magnitude. In the sky, the two planets will appear closer together for Prediction 2 in 1911 years than for Prediction 1 in 2 years. If we want an even 'tighter' alignment, we have to make the fractions for the orbit periods much closer to the actual periods of 88 and 224 days.