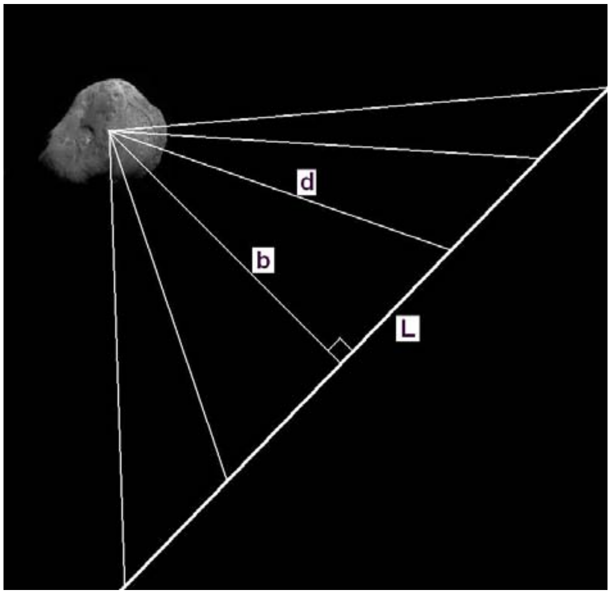
Deep Impact - Comet Flyby



On July 4, 2005, the Deep Impact spacecraft flew by the comet Tempel-1 along a path shown in the figure to the left, at a speed of V=10 km/sec. Its closest distance to the comet was b = 500 kilometers at a time, t=0. The distance traveled along the path is given by L = Vt.

The diameter of the comet is D = 8 kilometers, and the distance to the comet in kilometers is d(t), so the angular diameter of the comet in arcminutes is given by

$$\Theta(t) = 3438\frac{D}{d(t)}$$

Problem 1 - What is the formula for the distance to the comet from the spacecraft defined as *d(t)*?

Problem 2 - What is the formula for the angular diameter of the comet as seen from the spacecraft at any time, t, along the trajectory, defined by the variables V, b and L? Simplify the formula by defining two constants B = 3438L/b and c = V^2/b^2.

Problem 3 - What is the exact numerical formula for the rate-of-change in time of the angular size of the Tempel-1 as viewed by the spacecraft as it flys by?

Problem 4 - What was the angular diameter of Tempel-1 at the closest approach, and how fast will the angular size be decreasing when Tempel-1 reaches one-half its maximum angular size?

Answer Key

Problem 1 - What is the formula for the distance to the comet from the spacecraft defined as d(t)? Answer: Use the Pythagorean Theorem and the diagram to determine that

$$d(t) = \sqrt{b^2 + V^2 t^2}$$

Problem 2 - What is the formula for the angular diameter of the comet as seen from the spacecraft at any time, t, along the trajectory? Simplify the formula for $\Theta(t)$ by defining two constants B = 3438D/b and c = V^2/b^2. Answer:

$$\Theta(t) = \frac{B}{\sqrt{1+ct^2}}$$

Problem 3 - What is the exact numerical formula for the rate-of-change in time of the angular size of the Tempel-1 as viewed by the spacecraft as it flys by? Answer; Evaluate B and c for the specific values of D, b and V for the Tempel-1 flyby. Remember to use consistent units (all units in meters, and meters/sec). B = 3438 x 8,000 meters/(500,000 meters) so B = 55 arcminutes, and c = (10,000 meters/sec)^2/(500,000 meters)^2 so c = 0.0004 sec^{-2}. Then the formula becomes,

$$\Theta(t) = \frac{55}{\sqrt{1+0.0004t^2}}$$

where $\theta(t)$ will be the comet angular diameter in arcminutes. The rate-of-change of $\Theta(t)$ with respect to time is just its first-derivative, which we then evaluate for B=55 and c=0.0004. The formula will provide answers in units of arcminutes/sec:

$$\frac{d\Theta(t)}{dt} = -\frac{1}{2}B\left(1+ct^2\right)^{-3/2}(2ct)$$

or

$$\frac{d\Theta(t)}{dt} = -\frac{0.022t}{\left(1+0.0004t^2\right)^{3/2}}$$

Problem 4 - What was the angular diameter of Tempel-1 at the closest approach, and how fast will the angular size be decreasing when Tempel-1 reaches one-half its maximum angular size?

Answer: For t=0 we get $\theta(0) = 55$ arcminutes. To decrease by a factor of two its final diameter has to be 27.5 arcminutes, so $\theta(t)$=27.5, and solve for t to get **t= 86.6 seconds.**

From the derivative formula, and evaluating it at t=86.6 seconds, we find that
$d\theta/dt$ = **- 0.24 arcminutes/sec**.