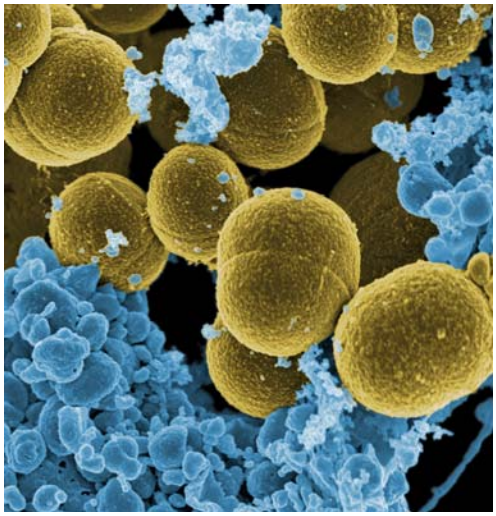
Estimating Maximum Cell Sizes



A simple living cell generates wastes from the volume of cytoplasm inside its cell wall, and passes the wastes out side its wall by passive diffusion.

If a cell cannot remove the waste fast enough, toxins will build up that eventually disrupt cellular functioning. The balance between waste generation and diffusion, therefore, determines how much volume a cell may have and therefore its typical size.

Photo: *S. aureus* bacteria escaping destruction by human white blood cells. (Credit: NIAID / RML)

Suppose the cell has a spherical volume, and that it generates waste at a rate of **a** molecules per cubic micron per second. Suppose it removes the waste through its surface by passive diffusion at a rate of b molecules per square micron per second, where 1 micron is 0.000001 meters.

Problem 1 - What is the equation that defines the rate, R, at which the organism changes the amount of its net waste products?

Problem 2 - For what value for the cell's radius will the net change be zero ,which means the cell is in equilibrium?

Problem 3 - A hypothetical cell metabolism is measured to be a = 800 molecules/μm^3/sec and b = 2000 molecules/μm^2/sec , about how large might such a cell be if it removed waste products only by passive diffusion?

Problem 4 - Two organisms are discovered that have a size of 1 micron and 10 microns. A) How does the ratio of their diffusion rates compare? B) If the surface waste diffusion rates are the same, how do their metabolic rates of waste production compare?

Problem 1 - What is the equation that defines the rate, R, at which the organism changes its net waste products?

Answer:

$$R = \frac{4}{3}\pi r^3 a - 4\pi r^2 b$$

Problem 2 - At what value for the cell's radius will the net change be zero?
Answer: Set R = 0, and then solve for r in terms of *a* and *b* to get:

$$\frac{4}{3}\pi r^3 a = 4\pi r^2 b$$

$$\frac{ra}{3} = b$$

$$r = \frac{3b}{a}$$

Problem 3 - A hypothetical cell metabolism is measured to be a = 800 molecules/μm^3/sec and b = 2000 molecules/μm^2/sec , about how large might such a cell be if it removed waste products only by passive diffusion?

Answer: r = 3 (2000/800) = 7.5 microns in radius.

Problem 4 - Two organisms are discovered that have a size of 1 micron and 10 microns.

A) How does the ratio of their diffusion rates compare? Answer: The smaller organism has 1/10 the ratio of the diffusion rates, b / a, as the larger organism.

B) If the surface diffusion rates are the same, how do their metabolic rates of waste production compare? Answer: If the surface waste diffusion rate, b, is the same, and the smaller organism has the lower ratio, then the smaller organism must have 10 times the waste production rate, a, if only passive diffusion is involved.