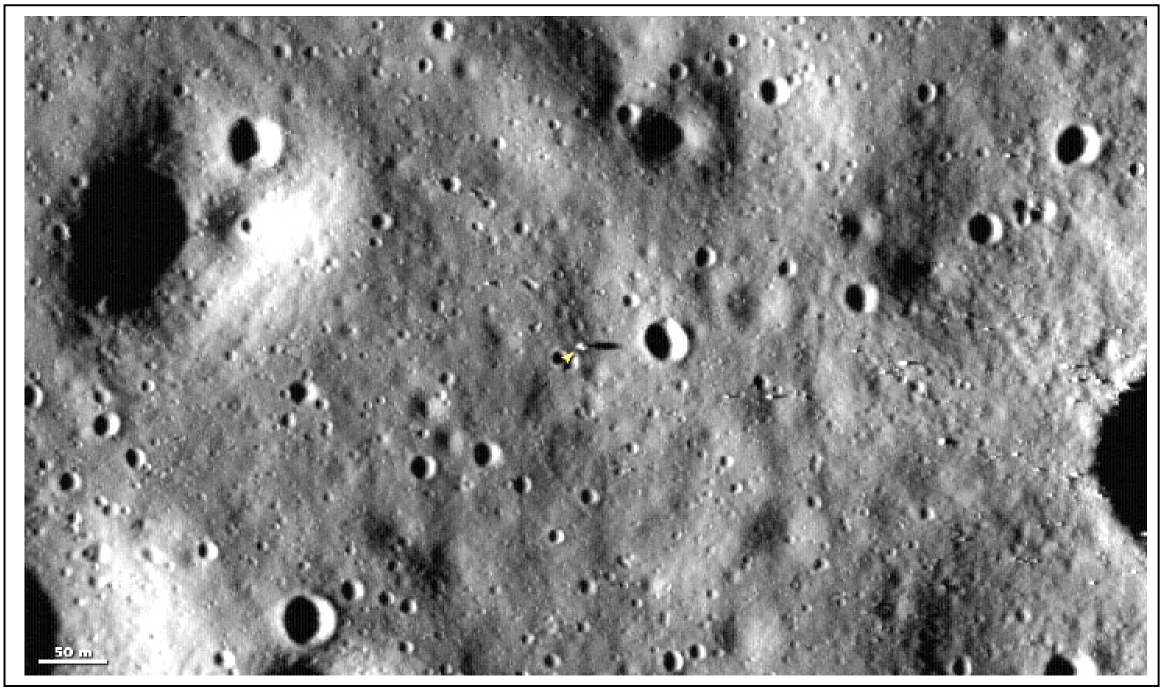
Lunar Crater Frequency Distributions



This image of the 800-meter x 480-meter region near the Apollo-11 landing pad (arrow) was taken by the Lunar Reconnaissance Orbiter (LRO). It reveals hundreds of craters covering the landing area with sizes as small as 5 meters. The Apollo-11 landing pad is near the center of the image, and is casting a long horizontal shadow to the right of the pad.

Astronomers use counts of the number of craters per kilometer^2 as a function of crater diameter to determine the age of a given lunar landscape, and the distribution of the sizes of the impactors. Crater counts are also used to determine which areas are safe to land. The power-law function below is based upon the above image from LRO and gives the surface density of craters near the Apollo-11 landing site in terms of craters per kilometer^2 of a given diameter, x, in meters. The range of validity is from 2 meters to 40 meters for this particular lunar area. Apollo-11 astronauts did not find any craters smaller than 2-meters near the landing area.

$$S(x) = 22000\, x^{-2.4} \text{ craters/km}^2$$

Problem 1 – Integrate the function S(x) to get the function N(x>m) which gives the number of craters per kilometer^2 with diameters greater than m-meters.

Problem 2 - Integrate the function S(x) to get the function N(x<m) which gives the number of craters per kilometer^2 with diameters smaller than m-meters.

Problem 3 – The Apollo-11 astronauts surveyed the area shown in the image above in order to find a landing site that was not part of a crater. To two significant figures, what is the maximum fraction of the area in the above image covered by craters larger than 2 meters in diameter? (Assume that the craters do not overlap, which is a good approximation to what the image shows.)

$S(x) = 22000\ x^{-2.4}$ craters/km^2

Problem 1 – Integrate the function S(x) to get the function N(x>m) which gives the number of craters per kilometer^2 with diameters greater than m-meters. Answer: The limits to the definite integral extend from m to infinity:

$$\int_m^{\infty} 22000x^{-2.4}dx \quad = \quad N(x>m)=\frac{22000}{1.4m^{1.4}}$$

Problem 2 - Integrate the function S(x) to get the function N(x<m) which gives the number of craters per kilometer^2 with diameters smaller than m-meters. Answer: The limits to the definite integral extend from 2 to m, because Apollo-11 astronauts did not see any craters smaller than 2 meters (x=2) in this area:

$$\int_2^{m} 22000x^{-2.4}dx \qquad N=\frac{22000}{1.4(2)^{1.4}}-\frac{22000}{1.4m^{1.4}} \quad \text{so} \quad N=5955-\frac{22000}{1.4m^{1.4}}$$

Problem 3 – The Apollo-11 astronauts surveyed the area shown in the image above in order to find a landing site that was not part of a crater. To two significant figures, what is the maximum fraction of the area in the above image covered by craters larger than 2 meters in diameter? Answer: S(x) gives the number of craters with a diameter of x per km^2. The maximum area occupied by these craters assuming that they are non-overlapping is given by $\pi (x/2)^2\ S(x)$. The total area covered by non-overlapping craters larger than 2 meters is given by the integral:

$$A=\int_2^{40}\pi\left(\frac{x}{2}\right)^2 22000x^{-2.4}dx$$

so $$A=\frac{22000\pi}{4}\int_2^{40}x^{-0.4}dx$$ then $$A=\frac{22000\pi}{4}\left(\frac{1}{0.6}\right)\left[x^{0.6}\right]_2^{40}$$

$$A=\frac{22000\pi}{4(0.6)}\left[40^{0.6}-2^{0.6}\right]$$

so that the cratered area is A = 28783 (9.14 - 1.52) = 220,000 square meters. The area in the image is 800 meters x 480 meters = 384,000 square meters, so the cratered area represents 100% x (220,000/384,000) = 57% of the surface area. So, the **maximum area that is covered by craters is 57%**. Note: That means that 43% of the area was safe to land on.