



The 'area under a curve' is an important mathematical quantity that defines virtually all mathematical functions. It has many practical uses as well. For example, the function plotted above, call it $P(X)$, determines the number of new planets, P , that were discovered each year, X , between 1994 through 2007. It was created by tallying-up the number of actual planet discoveries reported in research articles during each of the years. The actual curve representing the function $P(X)$ is shown as a black line, and the columns indicate the number of discoveries per year.

Problem 1 - How would you calculate the total number of planets detected between 1994-2007?

Problem 2 - What is the total area under the curve shown in the figure?

Problem 3 - Suppose $N(1995,2000)$ represents the number of planets detected during the years 1995, 1996, 1997, 1998, 1999, 2000. A) What does $N(1994,2007)$ mean? B) What does $N(1994,2007) - N(1994,2000)$ mean?

Problem 4 - Evaluate:

- A) $N(2002,2007)$
- B) $N(1999,2002)$

Problem 5 - Evaluate and re-write in terms of N (Example for a function defined for $X = A,B,C$ and $D : N(A,B) + N(C,D)$ is just $N(A,D)$)

- A) $N(1994,2001) + N(2002,2007)$
- B) $N(2001,2005) - N(2002,2005)$
- C) $N(1994,2007) - N(1994,2001)$

Answer Key

Problem 1 - How would you calculate the total number of planets detected between 1994-2007? Answer; You would add up the numbers of planets detected during each year, between 1994 and 2007. This is the same as adding up the areas of each of the individual columns.

Problem 2 - What is the total area under the curve shown in the figure? Answer; the total area is found by adding up the numbers for each column: $1 + 1 + 6 + 1 + 7 + 10 + 19 + 13 + 34 + 26 + 27 + 27 + 27 + 36 = \mathbf{235 \text{ planets}}$.

Problem 3 - Suppose $N(1995,2000)$ represents the number of planets detected during the years 1995, 1996, 1997, 1998, 1999, 2000.

A) What does $N(1994,2007)$ mean? Answer: It means the total number of planets detected between 1994 and 2007, which is **235 planets**

B) What does $N(1994,2007) - N(1994,2000)$ mean? Answer: It means to subtract the number of planets detected between 1995-2000 from the total number of planets detected between 1994-2007. $N(1994,2000) = 1 + 1 + 6 + 1 + 7 + 10 + 19 = 45$, so you will get $235 - 45 = \mathbf{190 \text{ planets}}$.

Problem 4 - Evaluate:

A) $N(2002,2007) = 34 + 26 + 27 + 27 + 27 + 36 = \mathbf{177 \text{ planets}}$.

B) $N(1999,2002) = 10 + 19 + 13 + 34 = \mathbf{76 \text{ planets}}$.

Problem 5 - Evaluate and re-write in terms of N:

A) $N(1994,2001) + N(2002,2007) = (1 + 1 + 6 + 1 + 7 + 10 + 19 + 13) + (34 + 26 + 27 + 27 + 27 + 36) = 58 + 177 = \mathbf{235 \text{ planets which is just } N(1994,2007)}$

B) $N(2001,2005) - N(2002,2005) = (13 + 34 + 26 + 27 + 27) - (34 + 26 + 27 + 27) = \mathbf{13 \text{ planets, which is just } N(2001, 2001)}$.

C) $N(1994,2007) - N(1994,2001) = (1 + 1 + 6 + 1 + 7 + 10 + 19 + 13 + 34 + 26 + 27 + 27 + 27 + 36) - (1 + 1 + 6 + 1 + 7 + 10 + 19 + 13) = 235 - 58 = \mathbf{177 \text{ planets which is just } N(2002,2007)}$