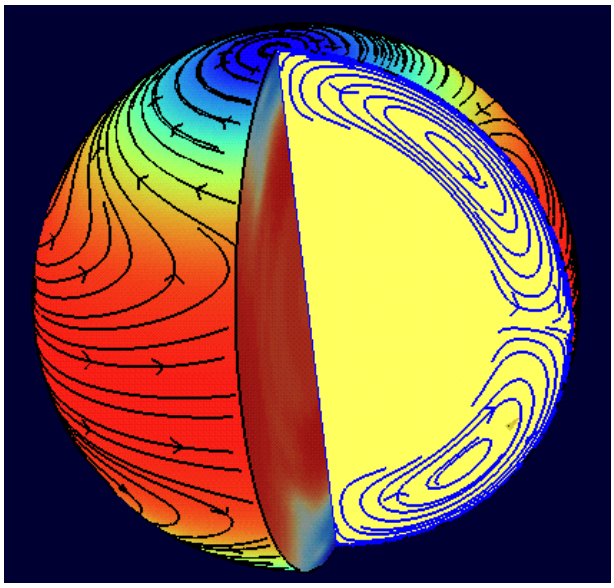

There are many situations in which differentiation has to be performed on formulae in astrophysics. Many objects such as stars and galaxies display 'differential rotation' which leads to many interesting and unusual phenomena. The formula describing these phenomena are usually 'differential equations' that relate changes in one quantity to changes in another.

Here are some popular equations used in astrophysics whose differentiation will test your basic skills!

Image: Model of solar differential rotation (Courtesy: Stanford Solar Center / NASA SOHO)

$$m = \frac{M}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$L = 4\pi R^2 \sigma T^4$$

$$R = \left(\frac{3 m V}{\pi \rho} t \right)^{1/4}$$

$$V = \frac{(Z+1)^2 + 1}{(Z+1)^2 - 1}$$

$$D = \left(\frac{k T}{4 \pi e^2 N} \right)^{1/2}$$

$$\Lambda = \frac{D^2 (D-1)^2 (D+2)}{8\pi (D+4)^2} m^2$$

Problem 1 - Find dm/dv - the rate of change of mass with velocity near the speed of light.

Problem 2 - Find dL/dT - The rate of change of a star's luminosity with its temperature.

Problem 3 - Find dR/dt - The rate of change of the size of an expanding supernova remnant with time (in other words, its expansion speed!).

Problem 4 - Find dV/dz - The rate of change of the apparent speed of a body with its gravitational redshift.

Problem 5 - Find dD/dN - the rate of change of the Debye shielding radius in a plasma with a change in the density of the plasma.

Problem 6 - Find $(1/m^2)$ $d\Lambda/dD$ - the rate of change of the energy of empty space as you change the number of dimensions to space.

Problem 1 - Find dm/dv - the rate of change of mass with velocity near the speed of light.

$$\frac{dm}{dv} = \frac{M v}{c^2 \left(1 - [v^2/c^2]\right)^{3/2}}$$

Problem 2 - Find dL/dT - The rate of change of a star's luminosity with its temperature.

$$\frac{dL}{dT} = 16 \pi R^2 \sigma T^3$$

Problem 3 - Find dR/dt - The rate of change of the size of an expanding supernova remnant with time (in other words, its expansion speed!).

$$\frac{dR}{dt} = 1/4 \left(3 mV/(\pi \rho)\right)^{1/4} t^{-3/4}$$

Problem 4 - Find dV/dz - The rate of change of the apparent speed of a body with its gravitational redshift. This requires using the quotient rule for differentiation d(U/V) = (1/V) dU - (U/v^2) dV

$$\frac{dV}{dz} = \frac{1}{(z+1)^2 + 1} 2(z+1) - \frac{1}{[(z+1)^2 + 1]^2} 2(z+1) = \frac{2(z+1)^3}{[(z+1)^2 + 1]^2}$$

Problem 5 - Find dD/dN - the rate of change of the Debye shielding radius in a plasma with a change in the density of the plasma.

$$\frac{dD}{dN} = -1/2 \left(kT/4\pi e^2\right)^{1/2} N^{-3/2}$$

Problem 6 - Find dΛ/dD - the rate of change of the energy of empty space as you change the number of dimensions to space. Again the quotient rule is needed where U = $D^5 - 3D^3 + 2D^2$ V= $D^2 + 8D + 16$

$$\frac{1}{m^2} \frac{d\Lambda}{dD} = \frac{5D^4 - 9D^2 + 4D}{8 \pi (D^2 + 8D + 16)} - \frac{[D^5 - 3D^3 + 2D^2] (2D + 8)}{[8 \pi (D^2 + 8D + 16)]^2}$$